

Vacuum Effects in the Presence of Walls

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Casimir effects have for many years been the subject of painstaking experiments and a variety of theoretical techniques, but it is only recently that they have attracted widespread attention. The surge of interest is attributable in large part to experimental developments. To the list of subtler reasons I would add one that can best be appreciated by the *cognoscenti*: Larry's lapidary elucidations of nearly every aspect of Casimir forces. I would also list a more personal if less important influence: some of us, before Casimir effects went mainstream, found encouragement in the fact that we could count Larry Spruch among the workers in the field.

The effect of walls is to impose boundary conditions on the electromagnetic field and thereby to modify its spectral properties. Thus, for example, the Lamb shift associated with vacuum field fluctuations in free space is modified in a way that depends on the position of the atom with respect to any walls. For an atom at a (large) distance d from a perfectly conducting wall, the position-dependent level shift is given by the Casimir-Polder formula: $E(d) = -3\alpha\hbar c/8\pi d^4$, where α is the static polarizability. For two parallel conducting walls in otherwise empty space the modification of the vacuum field leads to an attractive force between the walls given by the Casimir formula: $F = -\pi^2\hbar c/240d^4$ per unit area, where d is the wall separation. Both effects have in recent years been unambiguously demonstrated. Three excellent reviews of these and other Casimir effects are available ([1]- [3]).

Another wall effect occurs when the atom in the Casimir-Polder problem is replaced by an electron which, as a consequence of its electrostatic interaction with its image, can have a bound-state spectrum resembling that of a one-dimensional hydrogen atom. Shakeshaft and Spruch [4] showed that the electron experiences a level shift interpretable

in terms of its acquiring, “through vacuum fluctuations, a zero-point kinetic energy whose magnitude depends on the distance of the electron from the wall.” The coupling of the electron to the vacuum field in the half-space results in the interaction $V(d) = e^2\hbar/4\pi mcd^2$, where m is the rest mass of the electron and d is its distance from the (perfectly conducting) wall ([5], [6]). The radiative level shift for the ground state is relatively large and of order α , but becomes of order α^3 when a realistic model for a dielectric wall is considered [4]. These shifts have not yet been observed.

Another type of level shift is predicted to occur for an atom in a host dielectric medium ([7]- [9]). This effect will probably be difficult to observe, but let it be recorded that Spruch deserves the greatest share of credit for this prediction, and that the present author’s name appears first in Reference [8] only because of Larry’s well-known generosity. Aside from any experimental possibilities, this work clarifies the relation between the Lamb shift and the Casimir effect in terms of the change in the energy density of the vacuum field due to the bulk dielectric on the one hand, and the boundary of the finite dielectric on the other.

The theory of vacuum fluctuation effects is substantially more complicated in the case of dielectrics, as evidenced by Lifshitz’s treatment of the force between two dielectric walls separated by a third dielectric [10]. Indeed the theory can be fairly said to be *extremely* complicated. (Larry [11] and I [12] have both noted, as remarked by Ginzburg [13], that the Lifshitz formula is one of the few examples in the series of books by Landau and Lifshitz where a result is given without a derivation.) This is unfortunate in that it could discourage further studies of Casimir effects for more complicated geometries and for possible applications or implications in, say, nanotechnology.

Starting in a serious way with a paper by Spruch and Tikochinsky in 1993, approximate methods for deriving Casimir forces have been explored ([11], [14]). The Spruch-Tikochinsky paper relies on a combination of dimensional arguments and what I would call plausibility considerations to obtain reasonably accurate approximations (accurate to about 30% or better) for the Casimir interactions between an atom and a dielectric wall, an electron and a dielectric wall, and two walls.

More recently Schaden and Spruch ([15]- [17]) have described a novel semiclassical

method for the calculation of Casimir forces between conductors. Instead of adding contributions from all possible photon “paths,” they consider only those near periodic classical orbits. In the conventional approaches there are contributions to the Casimir energy from *all* closed periodic paths. This leads to well-known infinities which do not appear in the Schaden-Spruch theory. Their approach makes a connection between Casimir energies and ray optics, and appears to yield exact or very accurate results when one or more of the dimensions for a given geometry become sufficiently large that correspondingly large quantum numbers justify the semiclassical treatment. One of the successes of this semiclassical theory thus far may be found in the treatment of the Casimir interaction of two conducting spheres with radii R_1 and R_2 such that $1/d \gg 1/R_1 + 1/R_2$, where d is the closest separation of the spheres. The Schaden-Spruch semiclassical theory does not at present allow diffractive effects, but this may be possible in the same way that diffraction can be added in various degrees of approximation to geometrical optics.

Larry’s papers on Casimir effects are unusual in their combination of rigorous theory with shirt-sleeve physical explanations. They exude an air of intellectual curiosity and pleasure. I regret that I was not fortunate enough to know him earlier in our careers.

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This paper investigates further how the presence of a single reflecting plane wall modifies the usual Planckian forms in the thermodynamics of the massless scalar radiation in N-dimensional Minkowski... J.S. Dowker, R. Critchley, Vacuum stress tensor in an Einstein universe: finite-temperature effects. Phys. Rev. D. 15, 1484 (1977)ADSCrossRefGoogle Scholar. 14. R. Balian, B. Duplantier, Electromagnetic waves near perfect conductors, II. Casimir effect. It is possible for a classical field theory to have two stable homogeneous ground states, only one of which is an absolute energy minimum. In the quantum version of the theory, the ground state of higher energy is a false vacuum, rendered unstable by barrier penetration. There exists a well-established semiclassical theory of the decay of such false vacuums. In this paper, we extend this theory to include the effects of gravitation. Contrary to naive expectation, these are not always negligible, and may sometimes be of critical importance, especially in the late stages of the decay process. I. In this paper we consider the presence of the Wu-Yang magnetic monopole in the global monopole spacetime and their influence on the vacuum polarization effects around these two monopoles placed together. According to Wu-Yang [Nucl. Phys. **B107**, 365 (1976)] the solution of the Klein-Gordon equation in such an external field will not be an ordinary function but, instead, $\{it\}$ section. Because of the peculiar radial symmetry of the global monopole spacetime, it is possible to cover its space section by two overlapping regions, needed to define the singularity free vector potential, and to s