R Index of Some Bridge Graphs

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Abstract: *The degree of a vertex of a molecular graph is the number of first neighbors of the vertex. Sum degree and Multiplication degree of the vertex of a molecular graph is the sum of the degree of the vertices of the neighborhood vertices of the vertex and product of the degree of the vertices of the neighborhood vertices of a vertex respectively. The R degree of the vertex of a molecular graph is the sum of the sum degree of the vertex and the Multiplication degree of the vertex. The concept of degree in graph theory is closely interconnected to the concept of valence in chemistry. In this paper, some formulas are obtained for calculating the vertex based topological R index of the some Bridge graphs.*

Keywords: *Degree of vertex, Neighbourhood, Topological indices*

I. INTRODUCTION

A topological representation of a molecule is called molecular graph. A molecular graph is a collection of points representing the atoms and set of lines representing the covalent bondsin the molecule.The first degree based topological index was introduced by Randic in 1975. His index was defined as $R(G) = \sum_{uv} \frac{1}{\sqrt{d_u(G)d_v(G)}}$ [15].First and Second Zagreb indices, $M_1(G) = \sum_{v} d_v(G)^2$, $M_2(G) = \sum_{uv} d_v(G)d_v(G)$ [1,11]. Narumi- Katayama index, $NK(G) = \prod_{v} d_v(G)$ [9]. Multiplicative versions of the Zagreb indices $\prod_1(G) = \prod_{v} d_v(G)^2$, $\prod_2(G) = \prod_{uv} d_u(G)d_v(G)$, $\prod_1^*(G) = \prod_{uv} [d_u(G) + d_v(G)]$ [10,5,14]. Ernesto Estrada conceived the Atom-BondConnectivityindex, $ABC(G) = \sum_{uv} \sqrt{\frac{d_u(G) + d_v(G) - 2}{d_u(G)d_v(G)}}$ [7,8,12] .Augmented Zagreb index is the Modified version of *ABC* index, it was introduced by Furtula*et al.*[3]. It is defined as , $AZI(G) = \sum_{uv} \left(\frac{d_u(G)d_v(G)}{d_u(G) + d_v(G) - 2}\right)^3$ .Geometric-arithmetic indexwas invented by Vukicevicand Furtula[6]. It is defined as $GA(G) = \sum_{uv} \frac{\sqrt{d_u(G)d_v(G)}}{\frac{1}{2}[d_u(G) + d_v(G)]}$ .Zhang re-introduced the Harmonic index in [17], $H(G) = \sum_{uv} \frac{2}{d_u(G) + d_v(G)}$.Sum-connectivity indexwas introduced by Bo Zhou andNenadTrinajstić [4]. It was defined as, $SCI(G) = \sum_{uv} \frac{1}{\sqrt{d_u(G) + d_v(G)}}$ .The concept of R degree of a vertex and R index of a graph were introduced by the Author SileymanEdiz[18].The R degree of a vertex and R index of some well-known graphs given in[2].

II. DEFINITIONS

Throughout this paper, we consider only simple connected graphs, i.e. connected graphs without self-loops and parallel edges. For a graph G, V(G) and E(G) denote the set of all vertices and edges respectively. The degree of the vertex v is defined as the number of edges incident with v and denoted by d(v). The set of all vertices which are adjacent to vis called the neighborhood of vand denoted by N(v). For a vertex v, the sum degree of v is defined as $S_v = \sum_{u \in N(v)} \deg(u)$ and for a vertex v, the multiplication degree of v is defined as $M_v = \prod_{u \in N(v)} \deg(u)$ [18].Bridge is an edge of a graph whose deletion increases its number of connected components. Bridge graph (Tree) is a graph whose every edge is a Bridge. The subdivided star($K_{1,n}$:n) is a graph obtained as one point union of n paths of path of length two.[13].The $P_{n,p,k}$tree is a graph obtained from $P_n$by adding p neighbors to each of its nonterminal vertices and k neighbors to each of its terminal vertices[13].The Bistar$B_{n,n}$ is a graph obtained by joining the center (apex) vertices of two copies of $K_{1,n}$ by an edge[13].The Thorn rod $P_{p,t}$is a graph, which includes a linear chain of pvertices and degree-*t* terminal vertices at each of the two rod ends[13].The Corona $G_1 \Theta G_2$ of two graphs G_1 and $G_2$is defined as the graph G obtained by taking one copy of G_1 (which has nvertices) and n copies of G_2 and then joining the ithvertex of G_1 to every vertex in the ith copy of G_2. The graph $P_n \Theta K_1$ is called a comb. It is denoted as P_n^+ [13].The R degree of a vertex v of a simple connected graph Gis defined as $r(v) = S_v + M_v$. The first R index of a simple connected graph Gdefined as $R^1(G) = \sum_{v \in G} (r(v))^2$. The Second R index of a simple connected graph Gdefined as $R^2(G) = \sum_{uv \in E} [r(u) r(v)]$.The Third R index of a simple connected graph Gdefined as $R^3(G) = \sum_{uv \in E} [r(u) + r(v)]$.Our notation is standard and mainly taken from standard books of graph theory [16].

III. R INDEX OF SOME BRIDGE GRAPHS

Theorem:3.1

$R^1(K_{1,n} : n) = (2n + 2^n)^2 + n(16 + (2n+1)^2)$

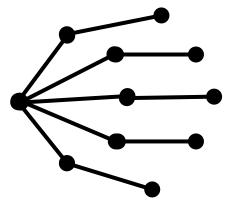
$R^2(K_{1,n} : n) = 2n(2n+1)(n + 2 + 2^{n-1})$

$R^3(K_{1,n} : n) = n(6(n+1) + 2^n)$

Proof: Let $|V(K_{1,n} : n)| = 2n + 1$ and $|E(K_{1,n} : n)| = 2n$

For the vertex v_1 , $S_{v_1} = 2n$, $M_{v_1} = 2^n$, For the internal vertices,

$S_{v_2} = \dots = S_{v_{n+1}} = n + 1$, $M_{v_2} = \dots = M_{v_{n+1}} = n$



$(K_{1,5} : 5)$

Fig 3.1

For the rest of the pendent vertices $S_{v_{n+2}} = \dots = S_{v_{2n+1}} = M_{v_{n+2}} = \dots = M_{v_{2n+1}} = 2$.

Hence $r(v_1) = 2n + 2^n$, $r(v_2) = \dots = r(v_{n+1}) = 2n + 1$, $r(v_{n+2}) = \dots = r(v_{2n+1}) = 4$

After Simplification,

$R^1(K_{1,n} : n) = (2n + 2^n)^2 + n(16 + (2n+1)^2)$

$R^2(K_{1,n} : n) = 2n(2n+1)(n + 2 + 2^{n-1})$

$R^3(K_{1,n} : n) = n(6(n+1) + 2^n)$

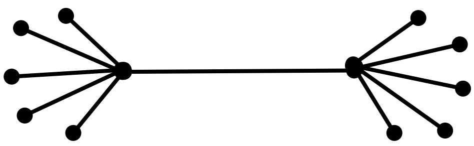
Theorem:3.2

$R^1(B_{n,n}) = 8n(n+1)^2 + 2(3n+2)^2$

$R^2(B_{n,n}) = (3n+2)(4n^2 + 7n + 2)$

$R^3(B_{n,n}) = (5n+2)(2n+2)$

Proof: Let $|V(B_{n,n})| = 2n+2$ and $|E(B_{n,n})| = 2n+1$



$B_{5,5}$

Fig.3.2

For the pendent vertices, $S_{v_1} = \ldots = S_{v_{2n}} = n+1,\ M_{v_1} = \ldots = M_{v_{2n}} = n+1$.

For the internal vertices v_{2n+1}, v_{2n+2}, $S_{v_{2n+1}} = S_{v_{2n+2}} = 2n+1,\ M_{v_{2n+1}} = M_{v_{2n+2}} = n+1$

Hence, $r(v_1) = \ldots = r(v_{2n}) = 2n+2$, $r(v_{2n+1}) = .r(v_{2n+2}) = 3n+2$

After Simplification,

$R^1(B_{n,n}) = 8n(n+1)^2 + 2(3n+2)^2$

$R^2(B_{n,n}) = (3n+2)(4n^2 + 7n + 2)$

$R^3(B_{n,n}) = (5n+2)(2n+2)$

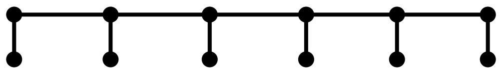
Theorem:3.3

$R^1(P_n^+) = 418 + 256(n-4) + 36(n-2)$

$R^2(P_n^+) = 81(n-5) + 54(n-2) + 132$

$R^3(P_n^+) = 18(n-5) + 15(n-2) + 56$

Proof: Let $|V(P_n^+)| = 2n$ and $|E(P_n^+)| = 2n-1$



P_6^+

Fig. 3.3

For the vertices $v_1, \ldots, v_n$ on the path P_n, $S_{v_1} = S_{v_n} = 4, S_{v_2} = S_{v_{n-1}} = 6, S_{v_3} = \ldots = S_{v_{n-2}} = 7$,

$M_{v_1} = M_{v_n} = 3, M_{v_2} = M_{v_{n-1}} = 6, M_{v_3} = \ldots = M_{v_{n-2}} = 9$,

For the pendent vertices $v_{n+1}, \ldots, v_{2n}$,

$S_{v_{n+1}} = S_{v_{2n}} = 2, S_{v_{n+2}} = \ldots = S_{v_{2n-1}} = 3,\ M_{v_{n+1}} = M_{v_{2n}} = 2, M_{v_{n+2}} = \ldots = M_{v_{2n-1}} = 3$

Hence, $r(v_1) = r(v_n) = 7$, $r(v_2) = r(v_{n-1}) = 12$, $r(v_3) = r(v_{n-2}) = 16$,

After Simplification,

$R^1(P_n^+) = 418 + 256(n-4) + 36(n-2)$

$R^2(P_n^+) = 81(n-5) + 54(n-2) + 132$

$R^3(P_n^+) = 18(n-5) + 15(n-2) + 56$

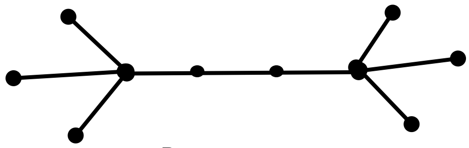
Theorem:3.4

$R^1(P_{p,t}) = 2(t+3)^2 + 2(3t+2)^2 + 64(p-4) + 8t^2(t-1)$

$R^2(P_{p,t}) = 2(3t+2)(t+11) + 4t(t-1)(t+3)$

$R^3(P_{p,t}) = 2(7t+5) + 16(p-5) + 6(t^2-1)$

Proof: Let $|V(P_{p,t})| = p + 2t - 2$ and $|E(P_{p,t})| = p + 2t - 3$



$P_{4,4}$

Fig.3.4

For the vertices $v_1,\ldots, v_p$ on the path P_p, $S_{v_1} = S_{v_p} = t+1, S_{v_2} = S_{v_{p-1}} = t+2, S_{v_3} = \ldots = S_{v_{p-2}} = 4,$

$M_{v_1} = M_{v_p} = 2, M_{v_2} = M_{v_{p-1}} = 2t, M_{v_3} = \ldots = M_{v_{p-2}} = 4,$

For the pendent vertices $v_{p+1},\ldots, v_{p+2t-3}$, $S_{v_{p+1}} = \ldots = S_{v_{p+2t-3}} = t, M_{v_{p+1}} = \ldots = M_{v_{p+2t-3}} = t$

Hence, $r(v_1) = r(v_p) = t+3, r(v_2) = r(v_{p-1}) = 3t+2, r(v_3) = r(v_{p-2}) = 8$

$r(v_{p+1}) = \ldots = r(v_{p+2t-3}) = 2t,$

After Simplification,

$R^1(P_{p,t}) = 2(t+3)^2 + 2(3t+2)^2 + 64(p-4) + 8t^2(t-1)$

$R^2(P_{p,t}) = 2(3t+2)(t+11) + 4t(t-1)(t+3)$

$R^3(P_{p,t}) = 2(7t+5) + 16(p-5) + 6(t^2-1)$

IV. CONCLUSION

In this paper, some formulas are obtained for calculating the new vertex based topological R index of the certain Bridge graphs.

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Every graph is an intersection graph of some family of subsets. Proof. Let G be a graph, and dene, for all v âˆˆ G, a set.Â When we repeat this process for all indices i with v1vi âˆˆ/ G for 2 â‰¤ i â‰¤ d + 1, we. obtain a graph Gâ€² as required. âŠ”âŠ“.Â DEFINITION. An edge e âˆˆ G is a bridge of the graph G, if Gâˆ’e has more connected components than G, that is, if c(Gâˆ’e) > c(G). In particular, and most importantly, an edge e in a connected G is a bridge if and only if Gâˆ’e is disconnected. On the right (only) the two horizontal lines are bridges. We note that, for each edge e âˆˆ G, e = uv is a bridge â‡â‡’ u, v in different connected components of Gâˆ’e . Theorem 2.3. An edge e âˆˆ G is a bridge if and only if e is not in any cycle of G. Proof. (â‡’) If there is a cycle in G containing e, say C = PeQ, then QP : v âˆ’â†’â‹† u is a. In graph theory, a bridge, isthmus, cut-edge, or cut arc is an edge of a graph whose deletion increases its number of connected components. Equivalently, an edge is a bridge if and only if it is not contained in any cycle. A graph is said to be bridgeless or isthmus-free if it contains no bridges. Another meaning of "bridge" appears in the term bridge of a subgraph. If H is a subgraph of G, a bridge of H in G is a maximal subgraph of G that is not contained in H and is not separated by H