Master's project
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Quantum thermometry and resource theories in generalized spin-boson models

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Project summary

Properties such as quantum coherence and entanglement are known to represent resources that can be used in metrology to surpass the experimental resolution of classical systems. This field has been attracting growing visibility in recent years due to the remarkable experimental progress in controlled quantum platforms. For this reason, quantum metrology is expected to be one of the first quantum technologies to fully transition from academia to market. However, several fundamental questions still remain open. In this project we propose to carry out a theoretical study on the possible uses of correlated quantum coherence in thermometry. We shall address the problem from the perspective of the recently developed resource theory of Distributed Coherence, which generalizes the notion of quantum coherence to multiple parties and therefore allows for a clean connection with notions such as quantum discord and Coherence Orders. Our main goal will be to single out what aspects of quantum coherence (local, non-local, etc.) are the ones ultimately responsible for a quantum advantage in this thermometric scenario. As a specific application, we shall study a generalized Spin-Boson model recently developed by Ms. Cipolla during her undergraduate studies. In this model an arbitrary number of qubits is allowed to interact with an arbitrary number of bosons, whose temperature we wish to measure. It therefore represents an ideal platform to understand how multiple qubits, and the structure of coherences among them, may be used to enhance the thermometric precision. It is our hope that this project will contribute both from an applied perspective, providing specific physical realizations of quantum thermometry, as well as from a fundamental perspective, as it will help elucidate which are the main resources that are ultimately consumed to provide quantum advantages.

1 Introduction

Properties such as the superposition of states (quantum coherence) and quantum entanglement are, in practice, extremely difficult to produce and manipulate in real systems. This is due to the contact between the system and the environment. While classical systems are somewhat easy to isolate, since it suffices to prevent energy loss, in the quantum realm the task is much more difficult as it also requires one to avoid leaks of information, which are much harder to plug [1]. This has been the primary drive of the experimental quantum physics community in the last decades: to isolate quantum systems well enough so as to enable for the coherent manipulation of these genuine quantum mechanical properties. For instance, in superconducting qubits, the coherence time (the time it takes for these quantum features to be lost to the environment) has increased more than 9 orders of magnitude in 15 years, from a few nano-seconds in the year 2000 to more than a full second nowadays.

This unprecedented progress naturally motivates the search for many applications involving the notion of quantum information processing. Examples include quantum communications, computing, simulations, metrology, thermodynamics and so on. A recent overview of this blooming field of research can be found in the European Community Quantum Technologies Roadmap [2]. In this project we shall be concerned with the use of quantum resources for metrology [3, 4]. The typical scenario is that of a probe composed of many particles, which is allowed to interact with a system of which one may extract a certain property. The interaction modifies the probe and these modifications encode the properties one wishes to measure. The precision of classical probes usually scale as $1/\sqrt{N}$, where N is the number of particles in the probe. Conversely, quantum systems can in principle achieve the so-called Heisenberg limit, with the precision scaling as $1/N$ [3].

From a theoretical perspective, there has been substantial progress in the description of quantum technologies in terms of *quantum resource theories*. Within this paradigm, one aims at identifying what is the precise quantum feature that gives a quantum advantage over a classical process. For instance, entanglement was long thought to be the ultimate resource for quantum computation and communication [5]. However, recent progress shows that quantum contextuality is also known to play a vital role [6]. Similarly, in the case of metrology, several approaches recently shown how quantum advantages can be obtained with other resources beyond entanglement. Examples include quantum coherence [7], hyperentanglement [8], Coherence Orders [9], radiation squeezing [10] and even spin squeezing in Bose-Einstein condensates [11]. A review on the recent routes for enhanced metrology beyond entanglement can be found in [4].

In this project we propose a theoretical study which will help advance this field of research in two specific and prominent directions, by combining *Thermometry* with *Distributed Coherence*. Quantum thermometry is a blooming application in metrology [12–15], motivated by the recent advances in the field of quantum thermodynamics, which is the field of expertise of Prof. Landi. A particularly interesting implementation of quantum thermometry was given recently in Ref. [13] using the spin-boson model [16]. In this model one qubit is allowed to interact with an arbitrary number of bosonic modes through an interaction that causes no spin-flips, but only decoherence. For this reason, this model can be solved analytically. And, as shown in Ref. [13], it offers a clean platform to study the basic properties of single-qubit thermometry. During her undergraduate studies (under a FAPESP scholarship 2017/20725-0), Ms. Cipolla has worked on a generalization of the spin-boson model to allow for an arbitrary number of qubits to interact with an arbitrary number of bosonic modes [17]. The interesting feature of this model is that it allows for the bosons to mediate an indirect interaction between the qubits. And, as we have shown in [17], this interaction is such as to allow for local coherence in the qubits to be converted into entanglement. Hence, it allows for an ideal platform for extending the thermometry results of Ref. [13] to the multi-qubit scenario.

On the other hand, we shall combine quantum thermometry with the recently developed resource theory of Distributed Coherence [18, 19]. The development of a resource theory of coherence [20] marks a paradigm shift in quantum resource theories, as it puts on firm grounds quantum coherence as the ultimate resource from which others can be derived (for a review, see [21]). The notion of Distributed Coherence [18,19] is a generalization of the traditional concept of coherence to the multipartite scenario, in which one focuses on the non-local part of quantum coherence. That is, the part which is shared among different parties. This naturally leads to a deep connection with the notions of quantum discord [22,23] and quantum entanglement [18,24]. These concepts also combine with other notions that may be of relevance to metrology, such as

Coherence Orders [9] and multilevel coherence [25].

Objectives

> In this project we propose to investigate the use of more general measures of coherence, such as Distributed Coherence [19] and Coherence Orders [9], as a resource for Quantum Thermometry. This will be done on both a formal, resource-theoretic level, as well as within a specific physical implementation based on a generalized spin-boson model developed by Ms. Cipolla in her undergraduate studies [17]. These results will serve to better elucidate what are the resources ultimately responsible for a quantum advantage in thermometry, as well as to the feasibility of the spin-boson model as a practical experimental implementation of a quantum-enhanced thermometer.

2 Methodology

We now proceed to list in detail the specific goals of this project and the methodology that the student will have to follow to complete each step. The description is divided into three main blocks. The first is a basic bibliographical review, together with the development of certain analytical and computational tools that will be required for the execution of the project. The second block contains the main research steps that Ms. Cipolla will have to progress in order to complete the basic objectives of the project. Finally, in the third block we list additional, optional routes, that she may explore if time permits.

This project will count with the expertise of several collaborators of Prof. Landi. Among them we mention Prof. Gerardo Adesso, from the University of Nottingham, which is already a collaborator under FAPESP project 2017/07973-5 and is a specialist, among other things, in quantum coherence and quantum correlations. Secondly, there will be a collaboration with Prof. Diogo Soares-Pinto from IFSC-USP and Dr. Diego Paiva Pires, from IIP-Natal, both of which are specialists in quantum metrology.

A. Bibliographical review and basic tools

A.1 **Foundations of quantum metrology:** As a starting point, Ms. Cipolla must familiarize herself with the basic notions of Quantum Metrology. This will be based on Ref. [26] as well as on established review articles such as [3, 4] and references therein. It will also count with the support of Dr. Diego Paiva Pires. Finally, Ms. Cipolla will also benefit from mini-courses that will be offered in IFUSP in 2019, organized by Prof. Landi, of which at least one will be focused precisely on Quantum Metrology.

A.2 **From metrology to thermometry:** In this part of the project, Ms. Cipolla will focus on studying the very recent developments in quantum thermometry, focusing on the recent review chapter in Ref. [14], which is part of the book "Thermodynamics in the quantum regime - Recent Progress and Outlook" of which Prof. Landi is one of the contributed authors.

A.3 **Calculation of the Quantum Fisher Information:** The central mathematical object in quantum metrology is the Quantum Fisher Information (QFI) matrix, which provides the ultimate bound on the precision of an experiment (known as the Cramer-Rao bound). Computing the QFI, however, is in general a difficult task, both analytically and numerically. Part of the preparation for the development of this project will, therefore, be based on developing a mathematical and numerical toolkit for computing the QFI. This will be based on strategies provided in Ref. [26], as well as a new approach put forth in Ref. [27].

B. Execution of the main objectives of the project

B.1 **Reproducing the results for quantum thermometry with a single qubit:** As the starting point for the actual execution of the research project, Ms. Cipolla with first reproduce the results of Ref. [13]. In this paper the authors considered the traditional spin-boson model as a platform for quantum thermometry. As one of the main goals of this project will be to study a generalized spin-boson model (to be described below), this paper will therefore serve as the basic ground on top of which Ms. Cipolla's research will be built on.

B.2 **Recasting the generalized spin-boson model for quantum thermometry:** During her undergraduate studies Ms. Cipolla developed the analytical solutions for a more general class of spin-boson models [17] in which N qubits (described by Pauli matrices σ_i^z) are allowed to interact with K bosons (described by annihilation operators b_k) with an energy conserving interaction. The basic Hamiltonian reads

$$H = \sum_{i=1}^{N} \frac{\Omega}{2}\sigma_i^z + \sum_{k=1}^{N} \Big\{ \omega_k b_k^\dagger b_k + f_k(\{\sigma_z\})(b_k + b_k^\dagger) \Big\}, \tag{1}$$

where $f_k(\{\sigma_z\})$ is an *arbitrary* function of all σ_i^z operators. The simplest choice would be a linear function of the form $f_k(\{\sigma_z\}) = \sum_{i=1}^{N} g_{ki}\sigma_i^z$. Thus, in this model the qubits do not interact directly, but only via the bosonic modes. The structure of the Hamiltonian (1) allows for great flexibility in choosing which are the best scenarios for thermometry. For instance, it describes a system with an infinite number of bosonic modes, as would be typical of a thermal bath, as well as a system with only $K = 1$ modes. This therefore allows one to study thermometry of finite-size environments and link it with notions of non-Markovianity. Similarly, the model (1) allows for the qubits to start in *arbitrary* initial states. Hence, it offers a platform to compare and contrast the efficiency gains from an entangled state to those from locally coherent states.

As the second step in this part of the project, Ms. Cipolla will therefore have to apply what she has learned in itens A.1 and A.2, to see which aspects of the general model (1) would be more interesting from a thermometric perspective.

B.3 **Computation of the QFI for 2 qubits:** As mentioned in A.3, the main mathematical object in metrology is the QFI, whose calculation is in general a difficult task. This is even more so if one is interested in analytical expressions. Motivated by this, we propose that Ms. Cipolla attempts to compute the QFI for two-qubits interacting with K modes, which would be the simplest non-trivial case in which one may observe a quantum advantage. This calculation will be contrasted with numerical estimates based on the techniques developed in A.3. From it, Ms. Cipolla will carry out a detailed analysis on the influence of the several parameters in the model. These include, as already mentioned in B.2., an analysis of different initial states for the 2-qubit system.

B.4 **Measurement strategies for 2 qubits:** The QFI provides the ultimate bound on the metrological precision. However, it does not provide any information on which kinds of quantum measurements on the qubits are necessary to achieve this bound. Motivated by the simplicity of this scenario, Ms. Cipolla will therefore study the amount of information that can be acquired when dealing with specific measurement protocols for 2 qubits. In particular, she will focus on the important question of local vs. non-local measurements, the latter being of course much more challenging to be implemented in the laboratory.

B.5 **Numerical analysis for N qubits:** After having understood in detail the case of 2 qubits, Ms. Cipolla will then carry out a numerical study of the QFI for N qubits. This will be based on the study carried out in A.3 on the possible strategies for studying the QFI. The main goal of this part will be to understand how the sought for quantum advantages scale with the number of qubits N.

B.6 **Resource theory of Coherence Orders:** Quantum coherence measures the amount of superpositions between different quantum states. However, not all superpositions will physically have the same importance. Take, for instance, the case of N spin 1/2 particles with a computational basis of the form $|\sigma_1, \ldots, \sigma_N\rangle$, where $\sigma_i = \pm 1$. Now define the total magnetization as $\mathcal{M}(\boldsymbol{\sigma}) = \sum_{i=1}^{N} \sigma_i$. It is then quite clear that, from a physical perspective, superposing two states with the same, or at least similar, values of $\mathcal{M}(\boldsymbol{\sigma})$ should be fundamentally different from a superposition of states with dramatically different values of $\mathcal{M}$. For instance, the extreme example would be a superposition of all spins up and all spins down. These two states are macroscopically different, so that the physics entailed by this superposition should in the end be fundamentally different.

In order to capture these effects, Prof. Diogo Soares-Pinto and Dr. Diego Paiva Pires, which will be collaborators in this project, have recently put forth a resource theory of Coherence Orders [9]. The basic idea is to divided and reclassify the usual resource theory of coherence into different orders of coherence measured, for instance, by the difference in magnetization (as described above). In this way the authors are able to identify the Coherence Orders as a natural resource for metrology in nuclear magnetic resonance.

In this part of the project we propose that Ms. Cipolla carries out a similar analysis within the context of the spin-boson model. To do so, she will consider the analytical solutions for the QFI obtained in B.3, together with numerical solutions for a larger number of qubits. One will then investigate whether the Coherence Orders can, in the end, be identified as the key resource to provide a quantum advantage in this spin-boson model.

B.7 **Resource theory of Distributed Coherence:** As a direct continuation of the previous goal, Ms. Cipolla will now investigate the notion of local vs. distributed coherence as recently put forth in Ref. [19]. From a general resource-theoretic perspective, the notions of Distributed Coherence and Coherence Orders, are of course related. However, this link has so far not been explored (this will be listed in Sec. C as one of the optional activities for Ms. Cipolla). In this part of the project, we propose that Ms. Cipolla studies the framework of Ref. [19] and try to apply some of the quantifiers that have been developed there to the specific context of the spin-boson model.

C. Optional routes of investigation

C.1 **Connecting Distributed Coherence with Coherence Orders:** As the first optional route of investigation, Ms. Cipolla may carry out a formal mathematical study attempting to connect the resource theories of Distributed Coherence [19] and the notion of Coherence Orders [9]. This study would be of great value, as it would be much more general in scope than the results aimed for in Sec. B. Notwithstanding, it would also be much more sophisticated and could very well be a topic for a PhD. That being said, however, we mention that the experience that Ms. Cipolla will gain in working with the concrete examples of the spin-boson model should give her an interesting perspective on this problem, which could very well make it viable during the masters. Or, perhaps, as a topic that starts during the masters but eventually is finished in a PhD.

C.2 **Bosonic thermometry:** As a second alternative route, Ms. Cipolla may investigate other models of thermometry. More specifically, in view of the expertise of Prof. Landi in continuous variable systems, Ms. Cipolla may study models in which the qubits are replaced by bosonic modes (in addition to the already present bosonic modes in the environment). These models have a natural appeal in quantum optics. In fact, Prof. Landi is already undertaking similar investigations in opto-mechanical systems, in collaboration with Prof. Pierre Luis de Assis, from Unicamp and Prof. Daniel Valente, from UFMT.

3 Experimental implementation

The topics discussed in this project have a strong appeal to the experimental community and can, in principle, be implemented in a variety of platforms. One, in particular, is nuclear magnetic resonance. We have already begun a collaboration with Profs. Ivan Oliveira and Roberto Sarthour, from the Centro Brasileiro de Pesquisas Físicas (CBPF) in Rio de Janeiro, to implement some of the aforementioned thermometry setups in a nuclear magnetic resonance setup. Miss Cipolla will actively participate on this collaboration, which has potential for significant scientific impact. The funding from FAPESP will help to partially finance the trips of Miss Cipolla to CBPF.

4 Final considerations

In this master project, we propose a study on the use of quantum resources to provide advantages to thermometry. Quantum thermometry lies at the boundaries between Quantum Information and Quantum Thermodynamics and corresponds to a new and exciting field of research. The proposal will deal with the analysis of a specific physical model encompassing all of the features necessary to understand the general problem at hand. For this reason, the student will first gain expertise within this concrete example, to then try to generalize the results in the form of a more general mathematical framework. The project will count with the collaboration of several researchers that are specialists in related fields of knowledge. Moreover, it will count with the full support of Prof. Landi as well as his entire research group. In light of the growing interest in this field of research, we expect that this project will produce 2 to 3 scientific papers in high-quality research journals during the two years of execution.

Finally, we also call attention to the fact the knowledge and the tools that will be developed by the student during this project extend well beyond this specific area of research. One of the key defining features of the Quantum Information community is the drive to seek a unified language and set of principles of broad applicability. In this sense, therefore, this project will contribute to put the student in contact with the current state-of-the-art in theoretical Quantum Information.

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3 Quantum Theory of Spin. In this section, we quantize the classical action for spin introduced in the previous section. This type of construction of quantum spins had actually been used in Haldaneâ€™s theory of anti-ferromagnetism in one-dimensional spin chain. 3.1 Allowed Values for J. We rst write J = jhÂ¯ for later convenience, S = jhÂ¯ cos Î¸Ï†Ë™dt. The spin-boson model has nontrivial quantum phase transitions at zero temperature induced by the spin-boson coupling. The bosonic numerical renormalization group (BNRG) study of the critical exponents Î² and Î´ of this model is hampered by the effects of boson Hilbert space truncation. Here we analyze the mean-field spin boson model to figure out the scaling behavior of magnetization under the cutoff of boson states N b . We find that the truncation is a strong relevant operator with respect to the Gaussian fixed point in 0 < s < 1/2 and incurs the deviation of the exponents from the class We consider spin-boson models composed by a single bosonic mode and an ensemble of N identical two-level atoms. The situation where the coupling between the bosonic mode and the atoms generates resonant and nonresonant processes is studied, where the whole system is in thermal equilibrium with a reservoir at temperature Î²âˆ'1.Â Second, we investigate the generalized Dicke model, introducing different coupling constants between the single mode bosonic-field and the environment, g1 and g2, for rotating and counter-rotating terms, respectively.Â The EPR-Bell correlations, and quantum entanglement in general, form the essential new ingredient which distinguishes quantum from classical information theory and, arguably, quantum from classical physics.