A Linguistic and Narrative View of Word Problems in Mathematics Education

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Jake and Jerry went on a camping trip with their motorcycles. One day Jerry left camp on his motorcycle to go to the village. Ten minutes later Jake decided to go too. If Jerry was travelling 30 mph and Jake traveled 35 mph, how long before Jake caught up with Jerry?

[Johnson, 1992, p.28]

A person bought oranges at the rate of 36 cents a dozen, had he received 6 more for the same money they would have cost him 6 cents a dozen less. How many did he buy?

[1902 Public School Leaving Examination of the Northwest Territories, in National Council of Teachers of Mathematics [NCTM], 1970, 419]

There is supposed a lawe made that (for furtheryng of tyllage) every man that doth kepe shepe, shall for every 10 shepe eare and sowe one acre of grounde, and for his allowance in sheepe pasture there is appointed for every 4 shepe one acre of pasture. Nowe is there a ryche shepemaister whyche hath 7000 akers of grounde, and would gladlye kepe as manye sheepe as he myght by that statute. I demaunde howe many shepe shall he kepe?

[Robert Record, The Ground of Artes, *1552, in Fauvel & Gray, 1987, 278]*

1,000 loaves of pesu 5 are to be exchanged, a half for loaves of pesu 10, and a half for loaves of pesu 20. How many of each will there be?

[Rhind Mathematical Papyrus, Problem 74, in Gillings, 1972, 130]

Per bùr [surface unit] I have harvested 4 gur of grain. From a second bùr I have harvested 3 gur of grain. The yield of the first field was 8,20 more than that of the second. The area of the two fields were together 30,0. How large were the fields?

[Babylonian document VAT 8389, Berlin Museum; translation from Van der Waerden, 1954, 66]

Mathematical word problems, or "story-problems", have long been a familiar feature of school mathematics. For many students, the "transformation" of word problems into arithmetic or algebra causes great difficulty, and a number of recent studies have addressed the linguistic and mathematical sources of that difficulty from a psychological point of view [Burton, 1991; Harel and Hoz,1990 ; Hoz and Harel, 1990; Mangan, 1989; Ormell,1991; Plane, 1990; Puchalska & Semadeni, 1987].

Recent curriculum policy documents such as, for example, the Cockcroft Report in Britain [Cockcroft, 1982, para 243, in Prestage & Perks, 1992] and the NCTM Standards documents in the United States [NCTM, 1989, 134 - 136] call for curricular relevance and an emphasis on the generic skills of problem solving. Many mathematics educators have interpreted these imperatives as merely a call for more, better, and more varied word problems. A few researchers [Borasi, 1986; Pinder, 1987; Brown & Walter, 1990] have begun to question the prevalent view that "problem-solving" means exclusively the solution of word problems, and studies in ethnomathematics [Lave, 1988; Lave, 1992; Nunes, Schliemann & Carraher, 1993] have revealed that people who are successful and efficient mathematical problem-solvers in "real-life" (i.e., life outside of school) may be unable to solve school word problems with pencil and paper, even when these word problems appear to be similar to "real-life" problems that the person is quite capable of solving.

Yet word problems are firmly entrenched as a classroom tradition, particularly in North American schools. They figure prominently in virtually all school mathematics textbooks and in the previous school experience of those who are now teachers and curriculum developers, and continue to be used unquestioningly by most teachers of school mathematics.

In this paper, I will try to establish a description of the mathematical word problem as a linguistic genre, particularly considering its pragmatic structure. Through a description of the pragmatics and discourse features of the genre, and through comparison of the word problems genre to other spoken and literary genres, I hope to find clues to the unspoken assumptions underlying its use and nature as a medium of instruction.

Previous linguistic studies of mathematical word problems

There have been a great many studies dating from the mid-seventies onward that have looked at mathematical word problems in terms of their "readability" (that is, the linguistic factors that make them easier or harder to read and understand) or students' ease or difficulty in translating them from "normal language" to mathematical symbolism. (See, for example, Burton [1991] and Nesher and Katriel [1977].) Although such studies use linguistics to analyze word problems, they presuppose the value of word problems as they are presently used in mathematics teaching and testing, and the need for students to become more proficient at solving them.

In this paper, I want to *problematize* the use of word problems in mathematics education. I will look at word problems as a linguistic and literary genre, and describe the features of that genre. I hope, by "making them strange", to enable mathematics educators to see word problems in a new way, to make the word problem genre a

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conceptual object that we will be able to circle around, look at from different perspectives, and compare usefully to other conceptual objects (for example, other literary and cultural genres).

Contemporary linguistics has gone through many changes as a discipline since its origins at the turn of the century, and many significant changes have taken place over the past fifteen years. Useful newly-linguistic areas such as pragmatics (the study of language in use in particular contexts), discourse analysis (the study of extended stretches of spoken or written text in terms of utterances and their relationships), sociolinguistics (the study of language and social class and power relationships), semantics (the study of the fields of word meanings), dialectology (the study of varieties of language, their distribution and history), stylistics (the study of literary texts in terms of their linguistic features) and genre studies (the study of "text types" in terms of their linguistic and contextual features) have recently become available to mathematics education researchers. (For example, Roth [in press] uses conversational analysis, a form of discourse analysis, to look at the sociology of math students' learning when dealing with "real-life" problems and word problems, and Tim Rowland [1992] and David Pimm [1987; 1995] apply new developments in pragmatics, particularly questions of *deixis*, or "pointing with words", to mathematics education.) In this study, I will use methods from pragmatics, discourse analysis, and genre studies to try to shed light on the nature of word problems as applications of mathematics and as stories. (Readers looking for definitions and examples of specialized terminology may wish to consult the appendix on "linguistics, discourse and genre analysis" at the end of this paper.)

Word problems and their three-component structure

You receive a paycheck worth $125.50. You must give 1/5 of your earnings to both the Provincial and Federal government. How much money do you have left?
(Word problem written by a grade 6 student in Vancouver, reported in Menon [1993]).

Most word problems, whether from ancient or modern sources, and including "student-generated" word problems, follow a three-component compositional structure:

1) A "set-up" component, establishing the characters and location of the putative story. (This component is often not essential to the solution of the problem itself.)
2) An "information" component, which gives the information needed to solve the problem (and sometimes extraneous information as a decoy for the unwary)
3) A question

Variations on this structure occur; for example, the set-up and information components are sometimes collapsed into one sentence by the use of subordinate clauses, or the information component and the question are collapsed into a single sentence by using a subjunctive "if . . then" structure.

Mildred Johnson addresses this structure explicitly in her interesting little instructional book, *How to solve word problems in algebra* [Johnson, 1992]. Her advice to students who are having trouble with the transformation of word problems into algebra includes the following:

> Look for a question at the end of the problem. This is often a good way to find what you are solving for. . . What you are trying to find is usually stated in the question at the end of the problem. . . Simple problems generally have two statements. One statement helps you set up the unknowns and the other gives you equation information. Translate the problem from words to symbols a piece at a time. [Johnson, 1992, 1 - 2]

Wickelgren [1974], in another book on mathematical problem solving, also identifies three parts to mathematical problems in general, and these fit closely with the three-part structure proposed for word problems:

> All the formal problems of concern to us can be considered to be composed of three types of information: information concerning *givens* (given expressions), information concerning *operations* that transform one or more expressions into one or more new expressions, and information concerning *goals* (goal expressions). [Wickelgren, 1974, 10]

In terms of the three components typical of word problems, the first component generally contains none of the above; the second component contains the *givens* and sometimes the *operations* (although the operations are often left unstated, since word problems are usually grouped together as a practice set for a particular set of algorithms that are currently the focus of classroom teaching. The choice of the correct algorithm from the currently-active set is considered one of the student's main responsibilities; to give the operation explicitly in the problem would be considered cheating). The third component, the question, identifies the *goals* of the problem.

The three-component structure of typical word problems seems, then, to be based on the structure of arithmetic algorithms or algebraic problems, rather than on the conventions of oral or written storytelling. In the case of an algebraic word problem, the student is required to write an algebraic equation in terms of a set of variables which are related to one another in a fixed (or "fixable") relationship that can be stated in terms of an equality or inequality. To "solve" the algebraic problem, the student must know which term to isolate (the *goal* or "the unknown"). This information is given (sometimes implicitly) in components two and three of algebraic word problems, which could be paraphrased as:

Component 2: The variables or quantities A, B, C, . . . are in the following relationship . . . (and you must deduce from the context of your lessons and the problem itself what operations are necessary to set up the equation).
Component 3: Solve for variable (X).

Component 1 of a typical word problem is, so far as I can see, simply an alibi, the only nod toward "story" in the story problem. It sets up a situation for a group of characters, places and objects that is generally irrelevant to the writing and solving of the arithmetic or algebraic problem

imbedded in later components. In fact, too much attention to story will distract students from the translation task at hand, leading them to consider "extraneous" factors from the story rather than concentrating on extracting variables and operations from the more mathematically-salient components 2 and 3

I think it is important to ask why the first component is included in word problems at all, or why this "translation" or "transformation" exercise should be considered important for one's mathematical studies. Many writers consider such problems to be practically useful, at least by analogy. Mildred Johnson writes:

> You will find certain basic types of word problems in almost every algebra book. You can't go out and use them in daily life, or in electronics, or in nursing. But they teach you *basic procedures* which you will be able to use elsewhere [Johnson, 1992, 1]

On the other hand, Thomas Kubala, in *Practical problems in mathematics for electricians*, makes direct claims for the usefulness of the problems he presents:

> The student learning electrical theory and wiring practices will find that by using PRACTICAL PROBLEMS IN MATHEMATICS FOR ELECTRICIANS his [sic] understanding of various mathematics principles will be reinforced because of their use in problems frequently encountered by an electrician . . Any student in a program of instruction in electricity will benefit from the use of this related problems workbook. Practicing electricians who desire to improve their math skills will also find it helpful [Kubala, 1973, 4]

It has been documented that the nature of the stories attached to the algebraic problems is relevant to students in terms of affect and in terms of the student's willingness to try to solve the problem at all [Sowder, 1989; Pimm, 1995] It is also interesting to note that, over the course of several years, students become enculturated in the world of school mathematics, and familiar with the conventions of the word problem genre to the extent that they are able to reproduce it. Ramakrishnan Menon [Menon, 1993] documents word problems in canonical form written by elementary school students who were asked to formulate their own mathematical questions. Menon also cites Ellerton's [1989] large-scale study of 10,000 secondary students in Australia and New Zealand who, when asked to write one difficult word problem, overwhelmingly wrote problems similar in form to those in their textbooks Jean Lave has noted,

> If you ask children to make up problems about everyday math they will not make up problems about their experienced lives, they will invent examples of the genre; they too know what a word problem is. [Lave, 1992, 77]

Puchalska and Semadeni [1987] comment that, while younger elementary school children often believe, naively, that the stories in story problems are relevant, more experienced older children know better:

> Radatz [1984] points out that, during problem solving, school beginners concentrate on stories rather than on numbers; during interviews it has been found that such children often augment the story with what follows from their own knowledge or experience Older children, however, always try to reach some solution, perhaps by a trial-and-error strategy, and they often believe that nothing is unsolvable in mathematics Children with little mathematical experience try to analyze the story more carefully, whereas older students have a specific attitude towards mathematics: It is viewed as an activity with artificial rules and without any specific relation to out-of-school reality [Puchalska & Semadeni, 1987, 10]

Word problems, intentions and speech acts: Locutionary, illocutionary, and perlocutionary force and uptake

Questions about the perceived purpose of the textbook writer or teacher in presenting word problems, or the student in solving them, relate to notions in pragmatics called *locutionary, illocutionary* and *perlocutionary* force, which are in turn part of speech act theory. J L Austin, the philosopher of language who originated speech act theory, saw the need for an analysis of language in the context of interactions, so that not only the literal meaning of an utterance but its meaning as action could be considered. Austin writes,

> We may be quite clear what "Shut the door" means, but not yet at all clear on the further point as to whether as uttered at a certain time it was an order, an entreaty or whatnot. What we need besides the old doctrine about meaning is a new doctrine about all the possible forces of utterances, towards the discovery of which our proposed list of explicit performative verbs would be a very great help [Austin, quoted in Levinson, 1983, 236]

Austin identified three kinds of speech acts that are simultaneously performed in an utterance:

> (1) a *locutionary act*: the utterance of a sentence with determinate sense and reference
> (2) an *illocutionary* act: the making of a statement, offer, promise, etc in uttering a sentence, by virtue of the conventional *force* associated with it (or with its explicit performative paraphrase)
> (3) a *perlocutionary* act: the bringing about of effects on the audience by means of uttering the sentence, such effects being special to the circumstances of the utterance. [Levinson, 1983, 236]

Levinson gives the example of the sentence:

> You can't do that

. which has a literal meaning (its locutionary force), and which may have the illocutionary force of protesting to the person being addressed, but the perlocutionary force of either checking the addressee's action, or bringing the addressee to his or her senses, or simply annoying the person.

Levinson differentiates between the perlocutionary force of an utterance, which is specific to the circumstances of issuance, and the consequences of an illocutionary act, which include the understanding of the illocutionary force

by the addressee(s). In the case of word problems, the perlocutionary force would include questions of affect in the individual learner (the intended or unintended effect of stimulating or boring, encouraging or discouraging, attracting or frightening or disgusting or delighting a particular learner, for example), and Levinson quotes Austin as admitting that perlocutionary force is often indeterminate or indeterminable. [Levinson, 1983, 237] However, the uptake, or understanding of the illocutionary force in word problems by learners, deserves further consideration.

Questions about the perceived purpose of the textbook writer in presenting word problems, or the student in solving them, relate to locutionary, illocutionary and perlocutionary force (that is, the literal meaning, the performative intention, and the effect upon the audience of an utterance). Applying this analysis to word problems poses some problems in terms of their locutionary force because of problems relating to deixis (the act of pointing with words). As discussed later (in the sections on verb tense and the Gricean maxim of quality), word problems do not generally have referents—that is to say, they do not refer to "real-life" objects, people, or places in any but the most arbitrary way. This could conceivably place word problems in the category of fiction, but I would argue that they are so deficient in the rudiments of plot, character, dramatic tension, poetic use of language, moral or social theme, etc., as to be very poor fiction at best. (The metaphor "word problems as parable" will be discussed later.) For the moment, I would prefer to view mathematical problems as a genre unto themselves, with *indeterminate* locutionary force.

Their illocutionary force seems to be quite directly accessible to students with sufficient enculturation in the genre; it is "Solve this!" or "Find X!" This command brings with it certain underlying assumptions:

- that "this" is solvable,
- that "X" can be found,
- that the word problem itself contains all the information needed to do this task,
- that no information extraneous to the problem may be sought (apart from conventional mathematical operations which likely must be supplied),
- that the task can be achieved using the mathematics that the student has access to,
- that the problem has been provided to get the student to practice an algorithm recently presented in their math course,
- that there is a single correct mathematical interpretation of the problem,
- that there is one right answer,
- that the teacher can judge an answer to be correct or incorrect, and especially,
- that the problem can be reduced to mathematical form—in fact, that the problem is at heart an arithmetic or algebraic formulation which has been "dressed up" in words, and that the student's job is to "undress" it again—to transform the words back into the arithmetic or algebra that the writer was thinking of, then to solve the problem.

Students' uptake, or understanding of the illocutionary force in word problems is, I think, quite clear, and could be paraphrased by a learner as follows:

> I am to ignore component 1 and any story elements of this problem, use the math we have just learned to transform components 2 and 3 into the correct arithmetic or algebraic form, solve the problem to find the one correct answer, and then check that answer with the correct answer in the back of the book or turn it in for correction by the teacher, who knows the translation and the answer

In this light, Puchalska and Semadeni's [1987] finding that children who were experienced in school math tried somehow to solve word problems which had missing, surplus or contradictory data is not at all surprising. I contend that these children had a well-developed schema with regard to word problems which included many facets of the genre, including its illocutionary force, and that a command to "make sense of this story in terms of everyday life" or to "search for deficiencies or contradictions in this problem" were never conceived as part of that illocutionary force (for the students, or indeed, for most teachers and textbook writers).

The question of "truth value"

The term "truth value" was introduced into semantics by Frege and Strawson, and was adopted from semantics into pragmatics. Frege wanted to be able to evaluate the meaning of all statements in terms of a principle of bivalence—that is, if something was not true it was false, and if not false, it must be true. (This principle is familiar to anyone working with mathematical proofs, where the "law of the excluded middle" allows for the possibility of proofs by contradiction.)

There are problems with Frege's notion of truth value, particularly when it is applied to utterances other than the propositional statements of philosophy. For example, questions, imperatives, and exclamations cannot be assessed for truth value. The truth value (if any) of statements in the context of fiction (storytelling, novels, plays, etc.) is also problematic. Lamarque and Olsen [1994, 54] give the following statements which might occur in a work of fiction: a) John worked in the fields; b) He found it tiring; c) There was once a young man who worked in the fields; d) Working in the fields is tiring—and consider that, while statements c) and d) might be assessed for truth value if construed outside of the context of the work of fiction (unlike a) and b)), this ignores the proper contextual construal of these statements. They give "a common alternative to the falsity thesis" in dealing with statements in fiction, which is the "no-truth-value" view of fictive utterance, partly attributed to Frege and Strawson. Three versions of this view are:

> 1. Sentences in works of fiction are neither true nor false because their (existential) presuppositions are false;
> 2. Sentences in works of fiction are neither true nor false because the sentences are not asserted;
> 3. It is inappropriate (mistaken, etc.) to ascribe truth or falsity to sentences in works of fiction. [Lamarque & Olsen, 1994, 57]

Lamarque and Olsen resort to a description of fictive utterances as pretence, or writing "as if" something were true, and distinguish three types of pretence: pretending to *be* ..., pretending to *do* ... and pretending *that* ... It is here that I can situate word problems; they pretend *that* a particular story situation exists. What is more, to paraphrase Lamarque and Olsen, readers of word problems must pretend *that* such a situation exists, *under instruction from the writer of the word problem*. Further, students "must pretend *that someone is telling them*" about that situation [Lamarque & Olsen, 1994, 71, authors' emphasis] The reader's response is not in terms of truth value but mimesis, yet at the same time the story is considered disposable, interchangeable with other equivalent stories, which would certainly not be the case with a work of fiction.

Linguistic and metalinguistic verb tense

Levinson [1983, 73 - 78] distinguishes between *linguistic tense* (L-tense) and *metalinguistic tense* (M-tense). By L-tense he means what is usually referred to as grammatical tense in a particular language; by M-tense, he means a semantic or deictic category of tense, which indicates the temporal location of an event relative to the coding time (CT) and/or receiving time (RT) of the utterance. (Levinson points out that, in the canonical situation of utterance, RT and CT are assumed to be identical, an assumption called *deictic simultaneity*.) In an M-tense system, we distinguish the temporal location of events in relation to CT: *past* refers to events prior to CT, *present* to events spanning CT, *future* to events succeeding CT, *pluperfect* to events prior to *past* events (which are themselves prior to CT), and so on. M-tenses are important in separating the deictic features of L-tenses from their modal and aspectual features. For example in English, L-future tenses always contain a modal element, and in a decontextualized sentence it is difficult to know just what balance of "futurity" and "intentionality" is indicated by modals like *will*, *should* and *may* in examples like the following:

> I will never go hungry again
> John should speak to her tomorrow.
> You may have visitors on Saturday morning

Some languages like Chinese may not have morphological verb tenses markers (and so may lack L-tense in this sense), and yet, as Levinson says, "we can confidently assume that there are no languages where part of an M-tense system is not realized somewhere in time-adverbials or the like, not to mention the implicit assumptions of M-present if no further specification is provided." [Levinson, 1983, 78]

Looking at examples of word problems from current British Columbia math textbooks, I found that determining M-tense in mathematical word problems is problematic. The difficulty is strongly linked to the lack of truth value in word problems—that is, their flouting of the Gricean maxim of quality. Although several patterns of L-tense typically appear in word problems, their M-tense seems to remain consistent. For example, in many word problems the first two sentences use L-present and the third L-future:

> A truck *leaves* town at 10:00 a.m. travelling at 90 km/h. A car *leaves* town at 11:00 a.m. travelling at 110 km/h in the same direction as the truck. At about what time *will* the car *pass* the truck? [Alexander *et al.*, 1989, 297]
>
> [A truck *leaves*: L- present
> A car *leaves*: L-present
> The car *will pass* the truck: L-future]

A second type uses L-past or L-present consistently in all three sentences.

> A ladder is unsafe if it *makes* an angle of less than 15° with a wall. A 10-m ladder *is leaned* against a wall, with the foot of the ladder 3 m from the wall. *Is* it safe? [Ebos *et al.*, 1990, 350]
>
> [A ladder *is*: L-present
> If it *makes*: L-present
> A ladder *is leaned*: L-present
> *Is* it: L-present]
>
> (This particular word problem is also interesting for its consistent use of passive, agentless sentences—there are no people in it.)

A great number of anomalies can be found which combine L-tenses in a self-contradictory way, that is, in a way that contradicts the usual use of L-tense in English, where the statements are assumed to have truth-value and the event is assumed to take place in a stable deictic relationship to coding time (CT):

> Each elephant at the Young Elephant Training Centre in Pang-ha, Thailand *eats* about 250 kg of vegetation in a day.
> *How much *would* 43 elephants *eat* in 1 day? 1 week?
> [Alexander *et al.*, 1989, 35]
>
> [Each elephant *eats*: L-present
> *43 elephants *would eat*: L-future
>
> If we accepted the truth of the first statement (and it certainly sounds convincing, since we're given the name and location of the Elephant Training Centre) we would expect "How much *do* 43 elephants *eat*" in the second sentence.]

I think that the most sensible interpretation of the unstable temporal, locational and personal deixis in these word problems is to interpret all of the above as *M-tenseless* and *non-deictic* (Levinson uses this analysis on such sentences as "Two and two is four" and "Iguanas eat ants", for example [Levinson, 1983, 77]), but having *conditional* or *subjunctive aspect*. That is to say, the word problems do not actually point to a person ("Jake", "Jerry" or "you"), place ("the Young Elephant Training Centre in Pang-ha, Thailand") or time (before, during or after CT). Since these are not real places, people, or situations, there is no absolute need for logical consistency in the use of L-tense (and L-tense is often used in ways that would be considered self-contradictory in standard expository English prose). Rather, word problems propose hypothetical situations with certain given conditions and ask for hypothetical answers. Most word problems could be rewritten in the form: "Suppose that (some certain situation A existed.) If (conditions B, C, D, ... held), then (what would be the answer to E)?"

The very inconsistency and seeming arbitrariness of L-

tense choices in word problems points not only to their M-tenseless and non-deictic nature, but also to an "understanding" between writer and reader that these supposed situations do not have truth value, and that the writers' intentions and the readers' task are something other than to communicate and solve true problems. (Otherwise the meaning of these problems in terms of a true situation would be very difficult to decipher.) This lack of truth value can be otherwise expressed as "flouting the Gricean maxim of quality".

Flouting the Gricean maxim of quality

It is my contention a feature of the word problem genre is a consistent flouting of the Gricean maxim of quality, which is to say that, as a genre, word problems have no truth value. This feature is intimately linked with, or perhaps a result of, their deictic indeterminacy. Time deixis, as shown through metalinguistic verb tense, has no referent. Personal deixis and place deixis (that is, the correspondence between the names of persons or places and their referents) have no truth value or are irrelevant. And yet the standardized form of the genre demands that declarative statements be made about these non-existent people, places and times. Such statements may be seen to be flouting the maxim of quality ("do not say what you believe to be false")

An example from a math textbook currently in use in British Columbia:

> Every year Stella rents a craft table at a local fun fair and sells the sweaters she has been making all year at home. She has a deal for anyone who buys more than one sweater. She reduces the price of each additional sweater by 10% of the price of the previous sweater that the person bought. Elizabeth bought 5 sweaters an paid $45.93 for the fifth sweater. How much did the first sweater cost? [Ebos *et al.*, 1990, 72]

The above could be reworded as follows without changing its truth value (although it would be a rather odd-looking word problem, highlighting as it does one of the implicit features of the genre:)

> Every year (but it has never happened), Stella (there is no Stella) rents a craft table at a local fun fair (which does not exist). She has a deal for anyone who buys more than one sweater (we know this to be false). She reduces the price of each additional sweater (and there are no sweaters) by 10% of the price of the previous sweater that the person bought (and there are no people, or sweaters, or prices) . . .

The hypothetical nature of word problems can be understood here, although it does not appear in the literal meaning of most examples of the genre. Again, Lave writes that "word problems are about aspects of only hypothetical experience and essentially never about real situations." [Lave, 1992, 78] This point is brought home in the cases where word problems seem to be referring to places, objects, or people known to exist, as in the following:

> A rock dropped from the top of the Leaning Tower of Pisa falls 6 m from the base of the tower. *If* the height of the tower is 59 m, at what angle does it lean from the vertical? [Ebos *et al.*, 1990, 354]

The tricky part of the story problem above is the "if" (my emphasis). Certainly the Tower of Pisa has been measured. Why use the conditional form here? Is it intended to indicate that the vertical height of the tower is not stable? (This may be true—it was recently closed to visitors because increases in its "lean" had made it dangerous . . .) Or is it a way of indicating that the referent for the words "the Leaning Tower of Pisa" is not the actual structure in northern Italy, but a hypothetical tower, or stick, or line segment, whose height could be set at any value (say, 59 m) and whose slope could be calculated using the given numbers and the Pythagorean Theorem? Again, the writer of the problem seems to be taking pains to say, "Here is a story, ignore this story"

Tradition: "I did them, and my kids should do them too"

All this leads me to a question for which I have no answer as yet, the question of the purposes of word problems as a genre. They are currently used as exercises for practicing algorithms, but such practice could certainly be achieved without the use of (throwaway) stories. The claim that word problems are for practicing real-life problem solving skills is a weak one, considering that their stories are hypothetical, their referential value is nonexistent, and unlike real-life situational problems, no extraneous information may be introduced. Nonetheless, they have a long and continuous tradition in mathematics education, and that tradition does seem to matter. Pinder [1987] makes a heartfelt case from a teacher's point of view for the non-practicality of word problems while acknowledging the strong pull of their tradition for the parent of one of her students. She is discussing the following well-known word problem still current in textbooks, which dates back at least to medieval Europe and probably to Roman times:

> A basin can be filled by three taps: the first fills it in sixteen hours, the second in twelve hours, and the third in eight hours. How long will it take to fill it when all are going together, it at the same time the basin is being drained by a pipe which can empty it in six hours? (Problem collected by Alcuin of York (circa 790 AD) as paraphrased in F.P. Sylvestre's *Traité d'arithmétique*, Rouen 1818.) [Plane, 1990, 69]

> A water tank has two taps, A and B. Line A on the graph shows how the tank drains if only tap A is open. Line B shows how the tank drains if only tap B is open.
> a) How long does it take to drain if only tap A is open?
> b) How long does it take to drain if only tap B is open?
> c) Use the graph to find out how long it would take to drain the tank if both taps were open. [Kelly *et al.*, 1987, 213]

Pinder writes of the father of one of her students who complained that his child wasn't being taught problems like the one above (which he had studied in school). She writes,

> On reflection I realized how very stupid it was to create a problem, to be worked out by manipulating symbols, about

> a situation which no one in their right mind would ever create. The problem was that if one filled a bath, pulled out the plug and left the taps running, how one could find how long it would take the bath to empty. My question was: what did it matter anyway? What possible *use* would the answer be? Could it be that one might need to know whether the bath might overflow and cause a flood? But if so, why not just turn off the taps? But perhaps they were stuck In that case surely it would be more useful for the children to learn how to turn off the water and how to locate the stopcock All in all, a pretty useless problem for children to work on; so why was that father worried that his child was not going to have to solve it? [Pinder, 1987, 74 - 75]

Word problems as parables?

I earlier discounted the idea of word problems as a fictional genre, citing their paucity of plot, character, human relationship, dramatic tension and so on. But what about David Pimm's suggestion that word problems be viewed as parables? [Pimm, 1995] In approaching this metaphor, I found both supporting and contradictory evidence in modern literary and theological theory that dealt with parable in other contexts.

A number of writers acknowledged the non-deictic nature of parable, a feature which we have seen in the word problem genre. J H. Miller writes that "all works of literature are parabolic, "thrown beside" their real meaning. They tell one story but call forth something else . . "Parable" is one name for this large-scale indirection characteristic of literary language, indeed of language generally." [Miller, 1990, ix] In an essay on Parable and Perfomative in the Gospels and in Modern Literature, he writes,

> Etymologically the word [parable] means "thrown beside", as a parabolic curve is thrown beside the imaginary line going down from the apex of the imaginary cone on the other side of whose surface the parabola traces its graceful loop from infinity and out to infinity again . it suggests that parable is a mode of figurative language which is the indirect indication, at a distance, of something that cannot be described directly, in literal language . Secular parable is language thrown out that creates a meaning hovering there in thin air, a meaning based only on the language itself and in our confidence in it The categories of truth and falsehood, knowledge and ignorance, do not properly apply to it [Miller, 1990, 135 - 139]

There is certainly the element of the indescribable involved in mathematical concepts, particularly those that deal with infinity, or with entities that exist perhaps only as mental images, and not in "this imperfect world" (an infinite straight line, a perfect circle, a point which has no part) Yet wouldn't the linguistic terms for these mathematical concepts (line, circle, point) be sufficiently "parabolic" deictic terms for these referents? Nonetheless, there is some appeal in the idea of story as a kind of homely way to refer to the indescribable, in the same way that religious parable speaks of spiritual matters in homely rather than theological terms Miller's reference to the irrelevance of truth-value for parables certainly could be seen to relate to to the word problem genre as well. Still, I am somewhat dissatisfied with Miller's fairly vague, general definitions; if the term parable can be taken to refer to all of language and literature and practically everything, what is its meaning?

Franz Kafka, in his book *Parables and Paradoxes*, briefly introduces his definition of parable:

> Many complain that the words of the wise are always mere parables and of no use in daily life, which is the only life we have. When the sage says: "Go over," he does not mean that we should cross to some actual place, which we could do anyhow if the labour were worth it; he means some fabulous yonder, something unknown to us, something too that he cannot designate more precisely, and therefore cannot help us here in the very least All these parables really set out to say merely that the incomprehensible is incomprehensible, and we know that already. But the cares we have to struggle with every day: that is a different matter [Kafka, 1961, 11]

Again, we can find resonance in the metaphor of word problem as parable, and Kafka's description could be read as the complaint of those like Pinder, who value real-life problem solving over the obscure references of word problems I'm sure many perplexed mathematics students believe that "all these word problems really set out to say merely that the incomprehensible is incomprehensible, and we know that already" But word problems differ from Kafka's parables in an important way—they stress the use of a correct method in order to arrive at a correct answer, while Kafka's wise one cannot suggest any course of action or any expressible goal. The illocutionary force of a word problem is an instruction to "Do this", "Solve this"; Kafka's storyteller is by no means so concrete or directive

Thomas Oden, writing about a collection of Kierkegaard's parables, asks,

> Why do we read Kierkegaard's parables, and why do they merit philosophical attention? Is it because they are like maddening puzzles daring some attempted solution? Is it because the problems they address drive to the depths of ordinary human experience? Or are they mere entertainment, revealing the comic side of human pretenses—subtle poetry, with virtually inexhaustible levels of meaning? Wherever the weight of the answer is to fall, anyone who lives with these parables for a while experiences both their power and their beauty Soon you realize that it is not you who are interpreting the parable but the parable that is interpreting you [Kierkegaard, 1978, ix]

Here I think the metaphor of word problem as parable begins to break down. Although word problems may be puzzles, and maddening ones at that, they do not dare but require solution—that is a large part of their force as speech acts. And I think it would be hard to argue that word problems, used as they are in our schools as "disposable" exercises, could be lived with over time, and seen to have inexhaustible levels of meaning, particularly poetic meaning about the depths of human experience. Nonetheless, there is the question of the durability of certain word problems, some of which appear to have been taught to scholars for centuries or millennia. Perhaps there

is something elemental or common to human experience in these, if we could find it, although perhaps their endurance simply speaks for the incredible conservatism of mathematical tradition.

Finally, in his book on the parables of Flannery O'Connor, John May quotes Dan O. Via, a modern hermeneutic scholar:

> For Via the parable in the narrow sense is "a freely invented story told with a series of verbs in the past tense" (e.g., the Prodigal Son, the Talents, the Unjust Steward). It is not concerned with the typical, but with "making the particular credible and probable." In the parable strictly conceived as fiction, "we have a story which is analogous to, which points to but is not identical with, a situation or world of thought outside of the story." [May, 1976, 14]

Thus far, the analogy with word problems holds up fairly well. The non-deictic use of verb tense, the concern with the credibility of particulars in a form that both does and doesn't refer to the particulars of the everyday world, all seem parallel. On the other hand, May writes,

> Inasmuch as the parable in a narrow sense thrives on the drama of human encounter as a figurative expression of the drama between God and man, it uses ordinary human language, rather than specifically theological terms, to mediate the ultimate reaches of reality to man. Reflecting the historical situation of their author, the parables proclaim what it means to exist in a boundary situation, how the eschatological crisis occurs within the confines of everyday existence. [May, 1976, 15]

Although word problems do reflect the historical situations of their authors, I think it would be stretching the metaphor rather far to claim that they "mediate the ultimate reaches of reality to man", that they involve eschatological crisis, or that they express the drama between human beings and God.

Perhaps in the exercise of viewing word problems as parables, we will be able to see word problems in a different way that will allow us to generate new ways of using them. I suggest, too, that delineating the boundaries of the word problem genre can allow us to play with those boundaries in interesting ways. In any case I do feel that it is important to think in new ways about the nature and purposes of word problems, about their inherent oddness and contradictions, and about our rationale for using them in school mathematics programs, rather than simply, unthinkingly visiting them upon future generations of schoolchildren.

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Appendix

A glossary of terms used from linguistics, discourse analysis and genre analysis

Deixis: from the Greek word for "pointing" (as in "index"), refers to the process of pointing with words Deixis studies the contextual referents for demonstratives ("this", "that"), pronouns ("I", "you", "it" etc.), verb tenses, context-referential adverbs of time and place ("then", "here") and "a variety of other grammatical features tied directly to the circumstances of utterance" [Levinson, 1983, 54]

Context of utterance is here referred to in terms of pragmatic indices, co-ordinates or reference points For example, the deixis of verb tense is analyzed with reference to the time an utterance was spoken or written ("coding time") and the time it was heard or read ("receiving time"), which may or may not be distinct. Demonstratives, adverbs of time and place and verb tenses are described on a continuum that ranges from closest ("maximally proximal") to furthest away ("maximally distal") from a central point of reference ("deictic centre")—for example, some dialects of American English have three adverbs referring to locations increasingly distal from a deictic centre ("here", "there" and "yonder").

Discourse analysis: This term has been adopted by a large number of disciplines, including various branches of linguistics, literary studies, ethnography, sociology, film studies and artificial intelligence. Generally, discourse analysis refers to the structural analysis of stretches of "text" (in its broadest meaning) at a level larger than the sentence or utterance. The texts in question may range from spoken discourse in classrooms or courtrooms, to written texts like stories, novels, poems, letters or graffiti, to dialogue in film or theatre, to oral genres like storytelling, speech making, gossip, jokes and puns (see Van Dijk, [1985a]; Van Dijk [1985b]; Coulthard, [1992])

The analytic methods that fall under the general heading"discourse analysis" are as heterogeneous as the texts they are used to analyze. Deborah Schiffrin [1994] writes that discourse analysis is "widely recognized as one of the most vast, but also one of the least defined, areas in linguistics" [p 5], and goes on to describe six approaches currently used in discourse analysis methodology: speech act theory, pragmatics, interactional sociolinguistics, ethnography of communication, conversational analysis and variation analysis.

Genre analysis: The term "speech genre" was coined by Mikhail Bakhtin to describe "relatively stable types of utterances" [Bakhtin, 1986, 60] The notion of genre analysis has since been taken up in other areas of cultural analysis, notably film studies and literary criticism.

Bakhtin stresses that genres can be analysed only by considering the whole of an utterance, including consideration of its thematic content, linguistic style (including lexical, syntactic and other grammatical features), its compositional structure, its expressiveness and its addressivity Since decontextualized words and sentences lose this quality of addressivity, a purely atomistic formal linguistic approach cannot capture the features of a genre.

Gricean maxims: The study of implicature has a basis in ideas expressed by H.P. Grice in a series of Harvard lectures in 1967 [Grice, 1975, 1978]. Grice looked for a set of assumptions underlying the efficient co-operative use of language. The five principles he found, including a general "co-operative principle" and four "(Gricean) maxims of conversation" are listed below:

1) *The co-operative principle*: Make your contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged

2) *The maxim of quality*: Try to make your contribution one that is true, specifically:

i) do not say what you believe to be false

ii) do not say that for which you lack adequate evidence

3) *The maxim of quantity*.

i) Make your contribution as informative as is required for the current purposes of the exchange.

ii) do not make your contribution more informative than is required

4) *The maxim of relevance*: Make your contributions relevant

5) *The maxim of manner*: Be perspicuous, and specifically:

i) avoid obscurity, ii) avoid ambiguity, iii) be brief, and iv) be orderly

Grice's point is not that all speakers must follow these guidelines exactly, since it is obvious that no one speaks this way all the time. Rather, he says that when an utterance appears to be non-cooperative on the surface, we try to interpret it as co-operative at a deeper level. Levinson gives the following example:

A: Where's Bill?

B: There's a yellow VW outside Sue's house
[Levinson, 1983, 102]

B's contribution, if taken literally, does not answer A's question, and it might seem as if B were being uncooperative and changing the topic However, if we assume that B is, at some deeper level, being cooperative and respecting the maxim of relevance, we try to make a connection between Bill's location and the location of a yellow VW, and conclude that, if Bill has a yellow VW, he may be at Sue's house.

Implicature: "provides some explicit account of how it is possible to mean more than what is actually said (i.e more than what is literally expressed by the conventional sense of the linguistic expressions uttered)."[Levinson, 1983, 97] For example, Levinson gives the following example:

A: Can you tell me the time?
B: Well, the milkman has come

... and paraphrases what native speakers would understand by this exchange as follows:

A: Do you have the ability to tell me the time of the present moment, as standardly indicated on a watch, and if so please do so tell me.
B: No I don't know the exact time of the present moment, but I can provide some information from which you may be able to deduce the approximate time, namely the milkman has come

Implicature studies the mechanisms by which speakers of a language can understand utterances' unstated relationship to context and to the speakers and listeners involved in the conversation (or to the writers and readers involved in a written exchange)

Pragmatics: In Anglo-American linguistics, pragmatics is often defined as "the study of language usage" [Levinson, 1983, 5]. This is a rather vague definition, and allows for an unintentional amount of overlap between pragmatics and other areas like sociolinguistics, psycholinguistics, etc. Levinson [1983] struggles with alternative, more specific definitions, and comes up with the following possibilities:

Pragmatics is the study of those relations between language and context that are *grammaticalized,* or encoded in the structure of a language [p 9]

Pragmatics is the study of the relations between language and context that are basic to an account of language understanding [p 21]

Pragmatics is the study of *deixis* (at least in part), *implicature,* presupposition, speech acts, and aspects of discourse structure [p 27]

Almost without exception, Western psychological theories have tended to cut experience into different and separate and often contrasting basics: thinking *or* feeling *or* acting; conation *or* cognition *or* affect; will *or* emotion *or* thought *or* perception. Since Old Testament times, since Plato and Aristotle, we have tended to believe that thought happens in one part of a person, feeling in another, will and action in another. For Kernberg, experience comes in wholes: any encounter provides an experience which has unity and leaves an integrated memory behind, though of course some parts of the experience may remain unconscious or preconscious or get repressed or remain unnoticed because the culture does not provide a word—a concept-label—for it

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Previous linguistic studies of mathematical word problems. There have been a great many studies dating from the mid-seventies onward that have looked at mathematical word problems in terms of their "readability" (that is, the linÂ In this paper, I want to problematize the use of word problems in mathematics education I will look at word problems as a linguistic and literary genre, and describe the features of that genre. I hope, by "making them strange", to enable mathematics educators to see word problems in a new way, to make the word problem genre a. 36. For the Learning of Mathematics 16,2 (June 1996) FIM Publishing Association, Vancouver, British Columbia, Canada. European research in mathematics education III. Rhetorical devices in mathematics classroom interaction: solving a word problem. Richard Barwell. University of Bristol, UK.Â Gerofsky, S. (1996) A linguistic and narrative view of word problems in mathematics education. For the Learning of Mathematics 16(2) 36-45. Latour, B. and Woolgar, S. (1986, 2nd edition) Laboratory Life: The Construction of Scientific Facts.