INVESTIGATING THE SPALART-ALLMARAS TURBULENCE MODEL

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Abstract

A brief introduction into turbulence modeling is made. The one-equation Spalart-Allmaras turbulence model which is used by the computational fluid dynamics research group of the University of Toronto Institute for Aerospace Studies will be introduced. Comparisons to other similar types of models and solution methods will be made.

Historical Perspective

The history of turbulence modeling can first be traced back to Leonardo da Vinci's drawings from the fifteenth century. Their qualitative insights demonstrated some of the fundamental issues involved with quantifying turbulence. Later work by Newton, Euler, Bernoulli, d'Alembert, Navier, Fourier, B. de St. Venant and Stokes led to a viscous fluid model with thermal conduction. In the past century, work by Reynolds, Prandtl, von Karman and Taylor finally led to a mathematical model for turbulent fluid motion based on the assumptions of continuum flow, averaged flow, viscous flow and the obedience to a set of turbulent postulates [6]. However, until the development of computers it was not feasible to obtain numerical results describing turbulence in a detailed manner. Furthermore, the power of computers has also been a limiting agent in achieving useful and accurate numerical results.

Computational Approaches to Modeling Turbulence

The key problem associated when dealing with turbulence is satisfying the need for a small enough computational length scale to ensure that turbulence is being modeled properly. Scientists and engineers are all too familiar with the exponential increase in computation when computational grids are refined. Only more recently, in the past 25 years, have computers enabled these researchers to employ Direct Numerical Simulation (DNS), to solve simple problems

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such as pipe flows. However, these solvers are limited to low Reynolds numbers. A simple code for a simple pipe flow would require 10e6 floating point operations per second and 10e9 words of storage [7]. While this is easily dealt with using today's computers, more complicated problems require computer memory and speed which can not be provided by even the most powerful of today's computers.

Anderson, et. al. [1] explain how Large Eddy Simulation (LES) is the most promising approach of recent technology. In this approach the smaller scales, which demanded so much computational power in the DNS approach, are treated by some sort of parameterization or model [7]. LES still provides three-dimensional, time-dependent results, but with a lot less computational effort. Furthermore, LES may still prove to be useful in difficult-to-handle, high Reynolds number flows [12].

A simpler approach in dealing with turbulence is to apply perturbations to the complete set of flow equations and then to enforce various degrees of closure on the newly-created perturbation terms. This closure can be achieved by two different methods. Either one can deal with how the Reynolds stresses are expressed, or one can deal with how the velocity and length scales are prescribed [12] and the Boussinesq assumption about the role of an eddy viscosity [1]. Reynolds stress models have various degrees of closure that can be made in order to compute the Reynolds stress tensor. The most complicated Reynolds stress model uses the full set of transport equations [12], resulting in a computational problem that deals with twelve transport equations! The former method dealing with the prescription of the various velocity and length scales and the Boussinesq assumption yields another set of classification which is commonly referred to as turbulent viscosity models [1]. Three distinct levels of closure in these models will be discussed, although intermediate and higher levels do exist. The three classes of models that will be considered are zero-, one- and two-equation turbulent viscosity models.

At the lowest level of turbulent viscosity models, one deals with zero-equation or algebraic models. These models assume that the turbulent field is at equilibrium with the mean flow. The turbulent length scale is then related to a some mean flow length scale such as the boundary layer thickness and the turbulent time scale to the mean flow time scale. Typical examples include mixing length models [12]. The key difficulty with these models is the scales chosen. The choice of a length scale such as a boundary layer thickness when such a thickness is not clearly defined is an alarming one [16]. At the next level, one deals with one-equation models. While these models still use the same ad-hoc assumptions as with the one-equation models, they use a transport equation to solve for the turbulent kinetic energy. Finally, at the highest level of the models being discussed, one deals with two-equation models. These models remove the ad-hoc assumptions made to determine the length and time scales by using two transport equations to solve for them. Examples of these models are k-ϵ and k-ω where k refers to the turbulent kinetic energy and the Greek letters ϵ and ω refer to dissipation and an inverse time scale respectively.

Several one-equation models are in existence, and only some are cited here for the interest of the reader: Bradshaw, Ferriss and Atwell (1967) [4], Wolfshtein (1969), Nee and Kovasznay (1969) [13], Norris and Reynolds (1975), Secundov, Smirnove, Kozlov and Gulyaev (1990) [14], Baldwin and Barth (1990) [2], and Spalart and Allmaras (1992) [15].

Motivation for the use of One-Equation Turbulence Models

Turbulent viscosity models, while being the simplest of the models described in the previous section, are more than adequate in solving most aerodynamic computational problems. Anderson et. al. [1] explain how turbulent viscosity models are by far the most popular models used by engineers, in the time their book was written. Therefore, only a motivation is made here for the use of a one-equation turbulence model, as opposed to a zero- or two-equation model. More accurate models are regarded as better, but not feasible with current technology, although Reynolds stress models are becoming increasingly popular [12].

When dealing with zero-equation or algebraic turbulence models, one runs into the problem of using boundary layer thicknesses which are not properly defined and the inaccurate prediction of shock-boundary layer interactions. This is clearly demonstrated when using the Cebeci-Smith [5] model and the Johnson-King [10] model (The Johnson-King model sometimes is referred to as a one-half-equation model). Other phenomena that zero-equation models have trouble dealing with, are massively-separated flows and ensuring the continuity of the eddy viscosity between an airfoil block and a wake block on an airfoil's computational grid. More importantly, unstructured grids are becoming more popular and zero-equation models can not be easily used on them. Where these models may hold promise in determining simple turbulent boundary layer calculations using multiple thin shear layers, other problems arise in determining the orientation of the grid lines. The bottom line for zero-equation turbulence models is they are very simple to code, they are inexpensive and they only work well for relatively simple problems.

While two-equation models ensure the simplest complete closure and are more mathematically sophisticated than one-equation models, they too have their disadvantages. For the higher computational effort, they provide no significant advantage over one-equation models for the prediction of shock-boundary layer interactions or separation from smooth surfaces [9]. In terms of 'higher computational effort', these models require much finer grids near walls, have much stronger source terms which degrade the convergence, and demand non-trivial upstream and freestream conditions for the turbulence variables [15]. Wall functions are usually introduced which further complicate these models [7]. The work of Godin et. al. [8] compares a one-equation turbulence model [15] to a two-equation turbulence model [11] in high-lift aerodynamic computations and concludes that each model has its own merits. However, as previously mentioned, the two-equation model is clearly more expensive.

These arguments do not prove that one-equation models are the best of the turbulent viscosity models. In fact, a compromise between the one-equation and two-equation turbulence models is ultimately the most preferred by current researchers who are using turbulent viscosity models.

The Development of the Spalart-Allmaras Turbulence Model

Other one-equation turbulence models were available for scientists and engineers to use in solving computational fluid dynamic problems, however, because of their early development, they were somewhat forgotten until only recently (i.e. in the past fifteen years). An example of this is the Nee-Kovasznay [13] turbulence model. It turns out that the Spalart-Allmaras turbulence model was developed from another turbulence model only two years its senior.

The one-equation Baldwin and Barth [2] model, which was developed in 1990, is an example of this compromise between the middle and highest levels of turbulent viscosity models. It has only one equation and is local, however it is derived from a two-equation k-ϵ model through some further assumptions. The simplification to a one-equation model is made by allowing a semi-local near-wall term. The mathematics will not be described here as this is supposed to be a short article and not a thesis! Being a one-equation model, the Baldwin and Barth model allows the programmer more control over its mechanics [15], while not including the complexity of a second equation.

The Spalart-Allmaras turbulence model [15] was first developed in 1992 and as described in their report, uses the Baldwin and Barth turbulence model as a framework for their model. The key modification made in this model is the approach used in determining the semi-local near-wall term. For various test cases and phenomena, such as free shear flows, near-wall flows at low and high Reynolds numbers and laminar-turbulent tripping regions the model produces acceptable results. Acceptable steady-state results were obtained, however in some shock-induced cases a limit cycle with a pulsating bubble was produced, whereas the algebraic models yielded steady solutions. To treat some of the acknowledged problems with the model, the developers suggested several improvements including the introduction of pseudo-compressibility terms, curvature effects and three-dimensional effects.

The Future of the Spalart-Allmaras Turbulence Model in the CFD Research Laboratory at UTIAS

In my discussion with Dr. David Zingg of UTIAS, some of the reasons for using the Spalart-Allmaras in his reasearch group were made vocal. First, he used the zero-equation Baldwin and Lomax model [3] as a reference and proceeded to explain how the Spalart-Allmaras turbulence model does not increase computational costs or slow down convergence, is more accurate for shock-boudary layer interactions, is marginally more accurate for mildly-separated flows and models wakes better. Next, Dr. Zingg explained in his research pertaining to high-lift aerodynamic computations [8] he found the two-equation Menter SST model [11] performed better for separated flows, while the Spalart-Allmaras model performed better for general computations for aerodynamic flows. The conclusion was that there was no significant benefit if a two-equation model was adopted.

As for the future of turbulent viscosity models in the CFD Lab at UTIAS, they will surely be around for some more time, because they provide reliable code for aerodynamic flows, however, Reynold stress models are preferred for internal flows such as curved pipes and ducts.

References

[1] Anderson, D.A., Tannehill, J.C. and Pletcher, R.H. *Computational Fluid Mechanics and Heat Transfer*, volume 1. Hemisphere Publishing Corporation, 1984.

[2] Baldwin, B.S. and Barth, T.J. A one-equation turbulence transport model for high Reynolds number wall-bounded flows. NASA TM 102847, 1990.

[3] Baldwin, B.S., and Lomax, H. Thin layer approximation and algebraic model for separated turbulent flows. AIAA Paper 78-257, 1978.

[4] Bradshaw, P., Ferriss, D.H., and Atwell, N.P. Calculation of boundary-layer development using the turbulent kinetic energy equation. Journal of Fluid Mechanics V28-3, pp.593-616, 1967.

[5] T. Cebeci and A.M.O. Smith. A finite-difference method for calculating compressible laminar and turbulent bou. Technical Report V92-3, Journal of Basic Enginnering.

[6] C.J. Chen. *Fundamentals of Turbulence Modeling*, volume 1. Taylor & Francis, 1998.

[7] Gatski T.B., Hussaini M.Y. and Lumley, J.L. *Simulation and Modeling of Turbulent Flows*, volume 1. Oxford Press, 1996.

[8] Godin, P., Zingg, D.W. and Nelson, T.E. High-Lift Aerodynamic Computations with One- and Two-Equation Turbulence Models. *AIAA*, 35(2):237–243, 1997.

[9] T.L. Holst. Viscous transonic airfoil workshop. compendium of results. *AIAA*, (91-0219), 1991.

[10] D.A. Johnson and L.S. King. A mathematically simple turbulence closure model for attached and separated turbulent boundary layers. Technical Report V23-11, AIAA Journal, 1985.

[11] Menter, F.R. Zonal Two-Equation k-ω Models for Aerodynamic Flows. AIAA Paper 93-2906, July 1993.

[12] Metais, O. and Ferziger, J. *New Tools in Turbulence Modelling*, volume 1. Springer, 1996.

[13] Nee, V.W. and Kovasznay, L.S.G. Simple phenomenological theory of turbulent shear flows. Physics of Fluids V12-3, pp.473-484, 1969.

[14] Secundov, Smirnova, Kozlov and Gulyaev. One-equation eddy viscosity model, 1990.

[15] Spalart, P. R. and Allmaras, S. R. A One-Equation Turbulence Model for Aerodynamic Flows. AIAA Paper 92-0439, January 1992.

[16] C. Walsh. Viscous flows and boundary layers. Course Lecture Notes, 2000.

Turbulence modeling has become one of the key problems in CFD. In aerodynamics, simple algebraic turbulence models have been widely used with fair success. However, the alge-braic model are not suitable for handling complex ow situations including ow separation, mul-tiple surfaces with turbulent regions near each. other, or wakes etc.Â To validate the functionality of the Spalart-Allmaras model the numerical method has been used to solve the turbulent at plate boundary layer and to solve the turbulent ow over a backward-facing step [7]. All the numerical results carried-out have been compared with theoretical, experimental and direct numerical simulations (DNS) data. The Spalartâ€“Allmaras model is a one-equation model that solves a modelled transport equation for the kinematic eddy turbulent viscosity. The Spalartâ€“Allmaras model was designed specifically for aerospace applications involving wall-bounded flows and has been shown to give good results for boundary layers subjected to adverse pressure gradients. It is also gaining popularity in turbomachinery applications.