DEPARTMENT OF ECONOMICS

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"OUR DAILY BREAD": MAURICE POTRON, FROM CATHOLICISM TO MATHEMATICAL ECONOMICS*

Christian BIDARD, Guido ERREYGERS and Wilfried PARYS

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Abstract. Maurice Potron (1872-1942) is a French Jesuit and mathematician whose main source of inspiration in economics is the encyclical *Rerum Novarum.* With virtually no knowledge in economic theory, he wrote down a linear model of production in which he formalized the notions of just prices and just wages. As early as 1911, he used the Perron-Frobenius theorem to prove the existence of a positive solution and established a duality result between the quantity side and the price side of the model. He returned to economics in the 1930s, but in both periods he failed to make a lasting impression upon economists.

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1. INTRODUCTION

In the harsh winter of 1942, with the German army occupying part of the buildings and requisitioning all available fuel, life was hard at the secondary school of Saint François-Xavier in Vannes, on the Atlantic coast of South Brittany (France). Too hard in any case for the French Jesuit mathematician Maurice Potron, who died there from pneumonia at the age of 69. Apart from a short obituary in the *Journal de la Société de Statistique de Paris* (Barriol, 1942) and in the school journal, his death went unnoticed.[1] After the end of the Second World War his name occurred only sporadically in the scientific literature, and his contributions seemed destined to be relegated to footnotes of specialized works of mathematics.[2]

In this article we draw attention to the highly original, but almost forgotten economic work of Potron, and show that he deserves a place in the history of economic thought. In two periods, from 1911 to 1914, and from 1935 to 1942, he presented and analyzed a model which integrates quantities and prices, and linked its solutions to the unemployment problem. The core of his work consists of a linear model of production of which the principal unknowns are the levels of production, employment, prices and wages. The importance of Maurice Potron for the history of economic analysis is apparent from the list of methods he used and the results he obtained from the very beginning of his investigations:

– Potron conceived a model of reproduction in which the productive system is represented by a matrix of technical coefficients and a matrix of labour inputs. The quantity problem consists in adjusting net production to the consumption by households, taking into account the interdependencies between industries. Potron clearly expressed many of the ideas which Leontief would develop later on, and he discussed the practical possibility of constructing an input-output table with heterogeneous labour. It should be noted, however, that Potron's technical coefficients are physical instead of monetary magnitudes, which allowed him to deal with the question of price and wage determination.

– In order to prove the existence of positive levels of output and employment satisfying a set of constraints (and similarly for prices and wages), Potron used the Perron-Frobenius theorem (Perron, 1907; Frobenius, 1908, 1909) in his model as early as 1911, and while doing so anticipated some aspects of Frobenius's (1912) generalization of the theorem. The importance of this theorem for economic theory has been acknowledged only after the Second World War.

– Potron showed that the existence of a solution depends upon a productivity condition, which is satisfied if and only if a crucial economic variable exceeds the dominant characteristic root of a nonnegative matrix. In Potron (1913, 1937c) he established that this condition is met if all principal minors of some associated matrix are positive, thereby anticipating the criterion which is now generally attributed to Hawkins and Simon (1949).

– Finally, Potron demonstrated that the solutions of the physical problem (output and employment levels) and of the value problem (price and wage levels) are intimately connected. Already in 1911 he stated a result which today would be qualified as a duality theorem. For disaggregated models, the priority of this type of result is usually attributed to von Neumann (1937).

These results shift the received chronology by a generation, and suffice to give Maurice Potron his due place among the great economists. Nevertheless, his work has remained virtually unknown until its recent rediscovery by Emeric Lendjel (2000, 2002) and the edition, in French, of a selection of his papers by Abraham-Frois and Lendjel (2004). This lack of recognition is partly related to the fact that Potron's interest in economic theory stemmed from an unusual source of inspiration for economists: the modernized version of the scholastic tradition put forward by the encyclical *Rerum Novarum* (1891), which updated the position of the Church on the 'social question'. Maurice Potron mobilized his scientific knowledge (as a mathematician and engineer) to the service of his religious convictions (as a Jesuit), in an attempt to derive the conditions of a just social order. He proposed to fight the social malaise

by putting into place the institutional and technical means which would facilitate the implementation of that order.

First we examine the life and work of Maurice Potron (Section 2). Then we study the theoretical aspects of his model (Section 3) and its implications for the social and economic organization (Section 4). We end with a short critique of his model (Section 5).

2. LIFE AND WORK OF MAURICE POTRON

We distinguish two main periods in the life and work of Maurice Potron, with the outbreak of the First World War as the turning point. The roots of his remarkable economic theory must be sought in his family environment, in his social network, and in the history of the Church and of France at the end of the 19th and the beginning of the 20th century. Circumstances relating to his person and to the circles he frequented help to explain why his views did not have an immediate impact when he formulated them.

2.1. The formative years

Maurice Potron's father, Auguste Potron (1844-1926), graduated as an engineer from the Ecole Centrale des Arts et Manufactures. In 1868 he married Cécile Frottin (1849-1918), the single daughter of a notary and previous mayor of the first district in Paris. Cécile Frottin's parents owned a castle at Courcelles, to the north of Paris, and a large building in the rue Saint-Honoré, 368 in Paris (first district), where the young couple went to live in one of the apartments. It was there that Maurice was born on 31 May 1872. To celebrate the happy event, his fervently Catholic parents ordered the construction, at Courcelles, of an 'asylum' which provided assistance to the poor and also included a primary school run by nuns.

Maurice was educated by private tutors, up to the level of Bachelor in Philosophy. He prepared his entry into a Grande Ecole at the Jesuit-run Lycée Sainte-Geneviève in Paris. In

1890, after a single year of preparation, he was admitted to the Ecole Polytechnique as the 91st ranked candidate. The Ecole Polytechnique, also known as 'X', is an élite military school which forms engineers. To complete their educational programme, the students end their formation in an applied school and, as the only one of his promotion, Maurice opted for a barely prestigious school (the so-called 'Poudres', i.e. Powder school). After a spiritual retreat in Clamart,[3] he enrolled in the Jesuit Novitiate of Canterbury in 1893. In those years the relations between the French state and the Catholic Church were especially tense (Mayeur and Rebérioux, 1984). The Ferry laws on free, compulsory and lay education targeted in particular the Jesuits, who were expelled from France, obliging them to move their religious formation abroad. This formation alternates periods of intellectual, philosophical and religious education, and periods of exercises and missionary work. Maurice took his First Vows in 1895, became a priest in 1905, and took his Solemn (or Final) Vows, which mark the full admission into the Society of Jesus, in 1912 (one of his brothers, Edouard, also became a Jesuit).[4] In between he taught mathematics at the Ecole Sainte-Geneviève (1896-1899), stayed at the Jesuit missions in Jersey (1899-1901) and Canterbury (1902), and finally returned to France (1903). This occurred in the anticlerical period of the Third Republic, a period which came to an end in 1905 when the law on the separation of Churches and State was voted. Although this law was regarded by the Church as a new assault on its rights, in the long run it contributed to the pacification of the religious quarrel.

In 1891, Pope Leo XIII's encyclical *Rerum Novarum* renovated the position of the Church on the social question, which had remained very conservative under the reigns of Gregory XVI and Pius IX. Either through his own reading, or through the actions of Jesuits like Henri-Régis Pupey-Girard (1860-1948) and Gustave Desbuquois (1869-1959) who introduced innovative missionary principles, the encyclical can be considered as the main source of reflection for Potron.[5] His economic model can be seen as a formalization of the principles expressed in *Rerum Novarum*. Of course the encyclical is not a treatise of economics, but

Potron was inspired by it when he defined the notions of just price and just wage and examined their compatibility. However, when the encyclical and Desbuquois only relied on the judgement of "prudent and upright men" (*Rerum Novarum*, §78) to determine justice in exchange, Potron arrived at the conclusion that the complexity of the economic problem prohibited prices from being determined by a simple decision of a committee of wise men. Instead, the assistance would be required of a "Bureau de Calculs" (1912a: 314(92)).[6] One can interpret the core of Maurice Potron's thoughts as a reflection on the mission and means of that Bureau.

Pupey-Girard had strong ties with the Potron family. For quite some time he lived in an apartment of Auguste Potron's Parisian building in the rue Saint-Honoré, and the numerous Catholic associations he initiated or firmly controlled (like the Union du Sacré-Cœur, and even the very active Ligue Patriotique des Françaises!) benefited from generous support of various types, including accommodation in several floors of the same building. His missionary zeal was especially directed towards the students who wanted to enter the Grandes Ecoles, then to engineering students of the Ecole Centrale, and later to those of the Ecole Polytechnique. The Union d'Ingénieurs Catholiques, which he founded in 1892 and later transformed into the Union Sociale d'Ingénieurs Catholiques (USIC), is historically speaking the first union of engineers in France, created at a time when their importance in society increased as a result of technical transformations. The USIC, which had a religious goal, served simultaneously as an association of mutual aid, an employment service and a centre of reflection on the role of the engineer, and was highly successful.[7] Its first president was Auguste Potron. Among other religious activities, the USIC organized spiritual retreats for engineers and others in Clamart, in Epinay sur Seine (Maurice Potron was there as the paymaster between 1907 and 1911), and then at the Villa Saint Régis at Mours (this villa had been bequeathed by Maurice's aunt).

To complete this panorama, we have to mention the social unrest in France at the beginning of the 20th century. Social legislation was hardly developed, but the 'social question'

was on the agenda as a result of numerous strikes and other actions organized by the trade unions, of both the reformist and the revolutionary camps (Mayeur and Rebérioux, 1984). The pressing importance of the social question explains not only the evolution of the Church, concerned as it was about the dechristianization of the labouring masses, but also the gradual adoption of social laws in France, such as the introduction of the first retirement funds, the Sunday as a day of rest, the reduction of the labouring day to 10 hours of work (while the trade unions campaigned for a working day of 8 hours), etc. The issue of the number of working hours or working days per year is also of central significance in Potron's model, as we shall see.

Potron used his mathematical knowledge to formulate and to solve his economic model. In 1904 he had obtained from the University of Paris a PhD in mathematics on a thesis entitled *Les Groupes d'Ordre* p^6. At that time one of the best specialists in finite group theory was the German mathematician Ferdinand Georg Frobenius (1849-1917).[8] Most probably because Potron already followed the work of Frobenius, he learned very early about Frobenius's theorems on positive matrices. By contrast, Potron mentioned Oskar Perron only in passing and even characterized him erroneously not as a German, but as a "Swedish mathematician" (1912a: 291(70)).

2.2. The construction of the economic model

On 15 October 1911 Maurice Potron published a short version of his work in the *Echo de l'Union Sociale d'Ingénieurs Catholiques*, the house organ of the USIC. The introductory note announced several other publications in which the results would be developed in more detail. The outline of the mathematical model and the statement of the main results were given in two notes submitted to the Académie des Sciences (1911b; 1911c). Already then Potron referred to a long mathematical discussion and extension of the Perron-Frobenius theorem, which he published in 1913 in the *Annales Scientifiques de l'Ecole Normale Supérieure*. A more

complete, but still largely non-mathematical version appeared in the 15 April 1912 issue of *Le Mouvement Social*, the "international catholic review" associated with the Action Populaire movement directed by Desbuquois.

This list of early publications indicates that Potron addressed two quite distinct audiences: on the one hand professional mathematicians, who were probably not very much interested in religious issues, and on the other devoted Catholics, who *a priori* had little inclination for formal models. But only exceptionally he found the right words for the non-mathematical reader. This happened in May 1912, when he published in the *Journal de la Société de Statistique de Paris* a short note on the "statistical researches that need to be undertaken", illustrated by data derived from a personal inquiry into the production process of bread.

An outlier in his early work is the article on the Taylor system of scientific management, published in the 1914 volume of *Le Mouvement Social*, in which Potron used his model to defend Taylor (according to the model, an increase of productivity makes the economic system feasible or facilitates the search for a solution). In comparison to his other publications, this article contains plenty of references, but most of them relate to the specialized literature on the Taylor system.[9]

In 1913 Potron started lecturing mathematics at the Institut Catholique d'Angers.[10] During the Great War he was first mobilized as officer in the Poudrerie du Bouchet, but he soon joined the artillery. He fought on some of the most dangerous battlefronts and got gassed, with a weakened health as a result.[11] On Christmas 1916 he received the *légion d'honneur*, and he ended the war in the service of the American army.[12] After the war he taught mathematics courses both at the Faculté Catholique de Lille[13] and at the Lycée Sainte Geneviève, which had moved from Paris to Versailles in 1913, where he remained until 1930. In that year he was appointed to the Institut Catholique de Paris, but without ever obtaining the official title of professor (Mgr. Baudrillart, the director of the Institute, had reasons to believe that Potron was

a good researcher but a poor teacher. Potron's reputation is confirmed by several witness accounts, and by the quality of his publications. It should also be kept in mind that he was hampered by a stutter.[14])

2.3. A new attempt in the 1930s

Until the mid 1930s Potron was fully occupied by his teachings, by the publication of collections of exercises, and by his mathematical researches on group theory and their applications to analysis. He was acknowledged as a competent mathematician (he was one of the representatives of the Société Mathématique de France at the International Congress of Mathematicians held in Oslo in 1936), but it should be noted that his principal collaborators in this field were Jesuits and that he often published in journals linked to the Grandes Ecoles and especially to the Ecole Polytechnique. The social network of Potron consisted essentially of Jesuits and of graduates of Polytechnique. The alumni were well represented at the Société de Statistique de Paris and at the Centre Polytechnicien d'Etudes Economiques (CPEE), also known as X-Crise (Brun, 1982: 19-35). Although economics was rather poorly developed at Polytechnique, the interest for it increased substantially in the interwar period. A case in point is that of Henry Le Chatelier (1850-1936): raised in a family of *polytechniciens*, this famous engineer and ardent Catholic prefaced the French translation (1911) of Taylor's *Principles of Scientific Management*. Le Chatelier also presented his own thoughts on social and economic topics, such as "L'organisation du travail et la question sociale" in 1929 and "La rationalisation et la crise économique" in 1930.[15] Closer to the X-Crise movement are the Guillaume brothers (Fischman and Lendjel, 2000) who published *Sur les Fondements de l'Economique Rationnelle* (1932) and *L'Economique Rationnelle* (1937). *L'Economique Rationnelle* is also the title of a book written by Divisia (1928), who taught economics at the Ecole Polytechnique.

The interest of the *polytechniciens* in economics reflected their conviction that it should be possible to extend the application of the scientific principles which they successfully used in

their engineering work to the machinery of the state. They had a solid command of the instruments of economic calculation, and they participated to the emergence of a system of national accounts, following Alfred Sauvy's example.[16] National accounting fulfilled a social need connected to the expansion of state intervention in the economy, through the creation of new institutions in response to the crisis of the liberal system. In combination with the Catholic influence in the circles of Polytechnique, this evolution provided strong arguments in favour of a rational organization of society according to principles which were not fundamentally different from the ones used in the organization of production. This technocratic standpoint, which seems to have been shared by Potron, surely conflicted with the tradition of pure economics, but at least it made it possible to introduce badly needed new concepts and instruments into the study of social and economic problems.

It is in introducing the discussion of a lecture by Jacques Branger at the *CPEE* in 1935 that Robert Gibrat (*polytechnicien* and part of the élite running the French state) referred to recent work by Ragnar Frisch (1934) and its use of matrices. This induced Potron to react and to revisit his old economic studies. He wrote a long letter to Gibrat in which he explained that some of the concepts reminded him of his own work more than two decades earlier: "You have talked about a matrix of supplies and demands considered by Frisch. There is another, very interesting matrix, which I think I have been the first to take into consideration" (1935: 62(127)).[17] The next year he presented his economic model at the International Congress of Mathematicians in Oslo (1936; 1937a), in a session co-organized by Frisch. Potron also organized a lecture series at the Institut Catholique de Paris (1937c) on the general theme "The mathematical aspect of certain economic problems". He announced these lectures in the *Bulletin du Centre Polytechnicien d'Etudes Economiques* (1937b) and in the *Bulletin de la Société des Amis de l'Ecole Polytechnique*, and in his private annotations he observed with satisfaction that François Divisia and René Roy, two *polytechniciens* turned economists, both attended the second and sixth lectures.[18] The attendance fluctuated between ten persons (for the

mathematical topics) and a hundred (for the lecture on the economic conclusions). Undoubtedly, not all of those who wanted to learn more about the economic and social doctrine of the Church were prepared to sit through long digressions on "recent discoveries in the theory of nonnegative matrices". However, Potron did make an effort at communication, which is rather exceptional for him, and constructed a numerical example which went a long way towards making his model understandable for his intended audience: he proposed to study a simple imaginary economy inspired by the Biblical story of the Hebrews' manna and asked to calculate the number of households required to collect the manna, collect the fuel, bake the cakes, or do nothing at all, as well as the economic characteristics of the associated regime (1937c: appendix (197-203)).

2.4. The failed diffusion of the economic model

In 1940 Potron moved to the secondary school of Saint François-Xavier in Vannes, where he died on 21 January 1942. In an obituary, Alfred Barriol (1942), the secretary of the Société de Statistique de Paris, summarized Potron's work and noted that he had actively participated in the meetings of the society. There is, however, hardly any trace that Potron's work had any impact at the time. A few years later, when the Jesuits asked one of their members to evaluate Potron's economic work, Michel Vittrant interpreted the 'manna' example literally and judged it ridiculous and contrary to the Bible.[19]

About half a century later, whilst doing research on the economists of Polytechnique, Emeric Lendjel rediscovered an offprint of Potron's first article of 1911. In between, the Perron-Frobenius theorem had become a standard tool in economic analysis. In collaboration with Gilbert Abraham-Frois, Lendjel has published a French edition of *Les Œuvres Economiques de l'Abbé Potron* (2004). Unfortunately, this publication contains a lot of editorial mistakes (typographical errors, bibliographical lacunae, etc.). Moreover, in their preface, the editors propose an interpretation of Potron's model which is so far removed from

his formalization that crucial concepts like unemployment or the number of hours worked per year have simply disappeared. This incomprehension of the model results in several errors, such as the use of a hybrid input matrix, the construction of which involves the multiplication of labour hours by consumption per worker per year (p. 39), a misunderstanding of the distinction between principal and secondary unknowns (p. 44), and the addition of vectors of different dimensions (p. 42).[20]

It should not be doubted that Potron wanted to tackle and to solve fundamental problems with regard to the causes of unemployment. His inadequate choice of publication outlets may be explained by his isolation from the community of economists: he conceived his model without having contact with contemporary economists and even without more than a superficial knowledge of the economic literature. Quite exceptionally Potron started one of his texts (1914a) with a brief citation from a "distinguished economist" (namely, Charles Gide), but he never repeated the experiment. His isolation transpires in the awkward notation he adopted (α for prices, ω for unemployment, etc.), and reveals itself fully in the use of expressions borrowed from everyday language instead of those adopted by the profession: *bénéfice* instead of *profit*, *résultat de travail* instead of *produit*, *économies* instead of *épargne* (for savings). In his works the terms *marché* (market) and *concurrence* (competition) are completely absent, in spite of the fact that the theory of prices is of crucial importance in his model. The startling statement that "prices, wages, employment levels […] are the result of *conventions* which, at least theoretically, are *free*" (1935: 63(129))[21] flies in the face of one of the oldest traditions of economic thought.

3. POTRON'S ECONOMIC MODEL

The economic model presented by Potron did not change significantly over the years. In the second period (1935-1942) Potron expressed labour time in hours instead of days. A more noteworthy change is that in the 1937 lecture series, including the appendix on the Hebrews'

manna, Potron assumed that the hourly wages are uniform per type of labour, independently of the firm, but later he occasionally mentioned that the assumption of non uniform wages may still be useful to solve certain problems (1937c: VI, §9 (195)). Our formalization follows the uniform wages version and introduces some other simplifications to shed light on the core of the model (the principle and the justification of these simplifications will become clear in section 4.1 below). The symbols used differ from those adopted by Potron, which vary and are counter-intuitive.

3.1. Satisfactory regimes

Potron characterized an *economico-social state* by three matrices C, L, A and a crucial number N.[22] The first matrix C relates to the typical consumption profiles of the different social groups: the annual consumption of a member of social group h ($h = 1, 2, ..., m$) consists of c_{hj} units of good j ($j = 1, 2, ..., n$), these data being summarized by the $(m \times n)$ matrix C of consumption baskets. A second set of constants describes the methods of production (one method per product), which are of the single-product type and admit constant returns. The production of one unit of good i requires l_{ih} hours of labour of social group h – which is identified with a profession – as well as a_{ij} units of good j ($i, j = 1, 2, ..., n$). These data are summarized by the $(n \times m)$ matrix L of labour input coefficients and the $(n \times n)$ matrix A of commodity input coefficients. Finally, there is the maximum number of working hours in a year: this number, N, takes into account the usual periods of rest and holidays, as given by law or tradition.[23]

The level of total consumption depends on the number of consumers in each social group. Several remarks are in order here. First, Potron does not consider the total number of consumers in each group as given. Second, the consumption basket is not individual but also meets the needs of the family. Third, each social group comprises both workers and non-

workers, whom Potron designated as "simple consumers" (e.g., 1911a: 6(59)), and once as *rentiers* (1914a: 166 et seq.). Fourth, the consumption basket is the same for any member of a given social group, whether he be a simple consumer, a full-time worker (who works N hours per year) or a partially unemployed worker (who works less than N hours).

In formal terms, let x_h be the number of labourers and s_h that of non-labourers of type *h*; for the economy as a whole these unknowns are summarized by the $(1 \times m)$ vectors x and s. Domestic consumption by labourers of all types amounts to xC, and by non labourers of all types to sC. Let y_i stand for the activity level of process *i*, which produces good *i*. The gross output vector of the whole economy is then equal to the $(1 \times n)$ vector y, and the vector of intermediate or industrial consumption equal to yA.

According to Potron, the economic system must obey certain fundamental principles. The principle of *sufficient production* states that enough is produced to meet industrial and domestic consumptions, i.e. the vectors y, x and s must be such that:[24]

$$y \geq yA + xC + sC \tag{1}$$

The *right to rest* stipulates that for each profession the yearly number of working hours does not exceed the maximum number of hours this profession can provide. This is expressed by the vector inequality:

$$Nx \geq yL \tag{2}$$

A system which satisfies both conditions (1) and (2) is called a *satisfactory regime of production and labour*.

Potron also contemplated the price side of the economy. Here Potron's approach is firmly rooted in the scholastic tradition, in the sense that the principles he advocated are meant to reflect the ideas of just prices and wages. The principle of *justice in exchange* states that the price paid for a good is at least equal to its cost ("Thou shalt not steal"). Let p be the

$(n \times 1)$ vector of prices and w the $(m \times 1)$ vector of hourly wages. Prices cover costs if the following holds:

$$p \geq Ap + Lw \tag{3}$$

The *right to life* is guaranteed if the annual wage earned by a labourer is high enough to enable him to buy the consumption basket corresponding to his social category. Assume first that all types of labourers work the maximum number of hours N. The right to life in this sense (1912a: 302(80)) is written:

$$Nw \geq Cp \tag{4}$$

However, the number of actual hours at work is often smaller than the maximum number *N*. Potron assumed that all workers of a given type *h* work the same number t_h of hours (therefore, no worker is totally unemployed, and underemployment, which is shared equally among all workers of the same type, is represented at the individual level by the gap between N and t_h). For activity levels y, the total number of working hours of type *h* amounts to $(yL)_h$, i.e. the *h*-th component of vector yL, and the number of labourers is x_h. The number of actual hours at work is therefore $t_h \equiv (yL)_h / x_h$, with $t_h \leq N$. Let T be the diagonal $(m \times m)$ matrix with the scalars t_h on the diagonal. The right to life in this stronger sense is expressed by the vector inequality:

$$Tw \geq Cp \tag{5}$$

In Potron's terms, a system which satisfies conditions (3) and (4) is a *simply* satisfactory regime of prices and wages, whereas a system which satisfies (3) and (5) is an *effectively* satisfactory regime of prices and wages.

3.2. The quantity problem and Perron-Frobenius

Having set up the model, Potron immediately understood that the principles he laid out may not be compatible when taken together. For instance, the principle of justice in exchange calls for

high enough prices with regard to costs, in particular to wages, whereas the right to life requires that wages are high enough with regard to prices (1912a: 290(70)). In the language of mathematics, the compatibility issue can be translated into the question of the existence of a positive solution to a system of inequalities. The problem can be split into two sub-problems.

Consider first the quantity problem, which concerns the existence of a satisfactory regime of production and labour: do there exist positive activity levels y, positive labour allocations x, and nonnegative non-labour allocations s such that the conditions of sufficient production and the right to rest are fulfilled? By taking advantage of inequality (2), the requirement on vector y in inequality (1) is written as $y \geq y(A + LC/N) + sC$, which for $s \geq 0$ implies $y \geq y(A + LC/N)$.

For economists, the most apparent novelty of Potron's works lies in his early application of the Perron-Frobenius theorem to an economic problem. For simplicity, we depart from Potron's hypotheses and assume that all matrices are indecomposable.[25] The Frobenius theorem asserts that a positive vector y satisfying the last inequality exists only if the dominant root of the nonnegative matrix $(A + LC/N)$ is at most equal to one (clearly enough, the condition is also sufficient: choose $s = 0$ and $x = yL/N$ in inequality (1)). In formal terms, the condition is written:

$$dom(A + LC/N) \leq 1 \tag{6}$$

Potron often referred to an alternative condition: for $s \geq 0$, inequality (1) implies that $y \geq xC(I - A)^{-1} \geq 0$, and the substitution in (2) leads to the inequality $Nx \geq xC(I - A)^{-1}L$, which holds for some positive vector *x* if and only if:

$$\nu = dom\left[C(I - A)^{-1}L\right] \leq N \tag{7}$$

The dominant root ν of the matrix $C(I - A)^{-1}L$ is what Potron calls the "characteristic number of the economico-social state" (1912a: 305-6(84)). The conditions (6) and (7) are equivalent,

but the economic interpretation in terms of a comparison between the magnitudes ν and N is clearer.

Let us begin by examining the borderline situation, where $\nu = N$. This situation lends itself to an interesting economic interpretation of the characteristic number. Consider the barest regime of production and labour which can be imagined: there are no simple consumers to be fed, not a single good is overproduced, and every labourer works exactly the maximum number of N hours per year. These conditions entail that we are confronted with a limit case of systems (1) and (2), with $s = 0$ and all previous inequalities relative to quantities turned into equalities. Then the labourers' total consumption basket is precisely equal to the net product they produce. Hence Potron's economic interpretation: the critical number ν stands for the "number of normal working [hours] that a labourer must provide so that the annual production obtained represents exactly the exclusive consumption of all labourers" (1913: 70(119)).

The case $\nu > N$ characterizes a system with profound economic difficulties: no solution can be found, no matter how the system is twisted or turned. The economy will be obliged, "par la force des choses", to the only way out: a change of the economico-social state (1912a: 308(87)). This can take the form of technical progress which decreases the characteristic number ν by increasing the efficiency of production or, alternatively, the form of an increase in N.

If $\nu < N$, by contrast, there is reason to let the church bells toll: infinitely many satisfactory regimes are possible. There are degrees of freedom in the number of workers and non-workers of various categories, the distribution of workers among industries or professions, the number of hours worked by each profession, and the surpluses produced. Not everything is allowed, though, because the actual activity levels or the actual distributions of workers and non-workers might not meet the inequalities (1) and (2). The difference with the previous case, however, is that now "the cause of the evil is not inherent to the economico-social state itself;

to remedy it, nothing needs to be changed to this state" (1912a: 310(88)): it suffices to change the proportions.

3.3. Prices and duality theory

The second problem examined by Potron relates to the existence of a satisfactory regime of prices and wages: does there exist a positive vector of prices *p* and a positive vector of hourly wages *w* such that the conditions of justice in exchange and the right to life are fulfilled? Two variants of the question can be distinguished.

Consider first the notion of a simply satisfactory regime: do there exist positive prices and wages solutions to the vector inequalities (3) and (4)? As inequality (3) implies $p \geq (I - A)^{-1}Lw$, substitution in (4) leads to the inequality $Nw \geq C(I - A)^{-1}Lw$, which holds for a positive wage vector w if and only if condition (7) holds. The remarkable property Potron obtains is that the existence of a satisfactory regime of production and labour is a necessary and sufficient condition for the existence of a simply satisfactory regime of prices and wages.

For economic theory and its applications, however, the significant notion is that of an effectively satisfactory regime (Potron ceased to refer to the concept of a simply satisfactory regime in his second period). Clearly enough, if the right to life is satisfied when workers are paid for their effective hours of work, it is also satisfied when they are presumed to be paid for the maximum number N of hours. Therefore, condition (7) remains necessary. A significant difference is that prices and wages now depend on quantities: an effectively satisfactory regime of prices and wages can be built only when the actual numbers of hours of work are known, i.e. after the determination of a satisfactory regime of production and labour. In such a regime, inequality (1) holds and the vector inequality (2) is written more precisely as the equality $xT = yL$ with $T \leq NI$. It follows that $y \geq yA + xC = y(A + LT^{-1}C)$, therefore the dominant root of the matrix $A + LT^{-1}C$ is at most equal to one, and there exists a positive price vector p such

that $p \geq (A + LT^{-1}C)p = Ap + L(T^{-1}Cp)$. The conclusion is that the price vector p and the wage vector $w = T^{-1}Cp$ are a solution to (3)-(5) and sustain an effectively satisfactory regime of prices and wages. In other words, an effectively satisfactory regime of prices and wages can always be grafted on a satisfactory regime of production and labour, which itself exists provided that the characteristic number of the system does not exceed the maximum number of hours of work. The results connecting quantities and prices are prominent in Potron's writings from the very beginning (1911c), and the fact that Potron introduced the rather artificial concept of a simply satisfactory regime as a step to facilitate the analysis shows that he was conscious of the importance of this duality result, in modern parlance.

Ironically, Potron's ignorance of economics has provided a double creative stimulus to his thinking. On the one hand, Potron was not hampered by any methodological considerations about the legitimacy of using mathematics in economics. He went much further than most of his contemporaries in the application of mathematics, and even invented new theorems aimed at the solution of economic problems. On the other hand, treating prices as pure social conventions which can be modified at will constitutes a framework of reflection which is favourable to the emergence of the concept of duality, since the question is then reduced to the existence of such prices. By contrast, any positive explanation of prices puts additional weight on the duality theory, viz. it raises the question of the market implementation of these 'ideal' dual prices.

4. THE ECONOMIC AND SOCIAL ORGANIZATION

4.1. Analysis of the social malaise

Potron's model is not only a tool for understanding the economic problems of his times, but also for conceiving a better social order and institutions which could solve the "economic malaise". This malaise manifests itself under two forms: unemployment on the one hand, and

prices and wages spiralling out of control on the other. Already in his earliest writings Potron showed awareness of the interdependence and possible tension between quantities and prices, as can be seen from the (translated) title of his note "Relations between the question of underemployment and those of just prices and just wages" (1912c).[26] A recurrent theme in his work is that similar looking phenomena could cover two types of crisis of a very different nature, which his model could identify and distinguish. The distinction relies on a comparison of the characteristic number ν of the economic system with the number N of legal working hours.

Consider the case $N < \nu$. At given wages, the principle of justice in exchange then requires a rise of prices, but this swells the costs of living to levels which endanger the principle of the right to life. At given prices, the right to life requires a rise of wages, but this endangers the principle of justice in exchange. A never ending race between prices and wages occurs, the only way out being a change of the economico-social state itself (in formal terms, a change of C, L, A or N).

In the other case, $\nu \leq N$, a less radical sort of crisis is possible, due to inappropriately chosen activity levels, amounts of workers and non-workers and prices and wages (in formal terms, bad choices of y, x, s, p or w). In this case too, a race between prices and wages may be set in motion (e.g. if firms stubbornly try to maintain their profit levels) (1935: 64(133); 1937c: VI, §8 (194)), but in principle this could be stopped by an internal adjustment because the system's fundamentals are sound. Even when the existence of satisfactory regimes (both for production and labour, and for prices and wages) is guaranteed, the search for such regimes is easier when the difference $N - \nu$ is large. If the difference is zero, there is a unique combination of satisfactory values (up to a factor of proportionality). If it is positive, the set of possible solutions grows with $N - \nu$. So the more the number of legal working hours exceeds the characteristic number of the economic system, the easier the system is manageable.

Potron did not express his model by means of the simple inequalities (1) to (5), but instead he opted for a system of equalities that are actually accounting identities. For example, instead of our inequality (1), he wrote down a mathematical equality of physical quantities:

> production = intermediate consumption + final consumption by workers and non-workers + overproduction

and similarly for monetary magnitudes:

> price = physical cost of production + wage costs + benefits
>
> income of a worker of type h = cost of living of this worker + economies

where overproduction, benefits (profits) and economies (savings) are additional nonnegative unknowns. In this way the number of variables is much higher than in our formalization and the set of solutions admits a number of degrees of freedom. Potron introduced a distinction between "principal" and "secondary" unknowns. The principal unknowns are "production, the allocation of workers, prices and wages" (1935: 63(130)), whereas the secondary unknowns are the components of the vectors of the non-workers, unemployment, overproduction, benefits of the firms and economies of the workers. Potron considered the distinction as crucial: "To sum up, all evil comes from the failure to recognize the distinction between principal unknowns and secondary unknowns." (*ibid.*, 64(133)) The grounds for the distinction lie in the set of values that the unknowns can take when the condition $\nu \leq N$ holds:

> (...) you can give *absolutely arbitrary* positive values to *all secondary unknowns*; for the principal unknowns, there will follow positive values such that everything goes at best. But you cannot give *absolutely arbitrary* values to the principal unknowns such that everything goes well. Despite this warning, one stubbornly persists in giving arbitrary values to the principal unknowns, because it is easier. It is very likely that one does not draw one of the favourable combinations. (*ibid.*)

Our formalization (1) to (5) follows Potron's distinction and emphasizes it by drawing attention to the principal unknowns only and ignoring the secondary unknowns (except the non-workers vector s: the reason for this particular treatment will appear in section 5). This led us to replace Potron's accounting identities by inequalities.

The fact that the crucial issue is the determination of the principal unknowns does not mean that secondary unknowns are uninteresting. By combining his accounting identities, Potron emphasized that "the yearly sum of the firms' benefits and the workers' economies represents exactly the yearly total cost of living of the non-working consumers" (1937c: II, §5 (155)).[27] Without doubt, Potron's interest in this question was stimulated by his family background, many of his relatives being rentiers.

Is there a chance that the economic system spontaneously reaches a satisfactory solution? This is a central question in economics, involving a study of the market mechanisms that respond to disequilibria with respect to prices or quantities. Manifestly, Potron had a limited knowledge of these writings, and his scepticism about the working of economic laws was sometimes explicit: "One often speaks about economic laws, rather vague for that matter, which revenge themselves on those who ignore them. I believe that the omission of the six systems of equations [Potron's accounting identities] is much more dangerous" (1935: 64(133)).

Because satisfactory values (of the principal unknowns) do not develop spontaneously, an economic malaise often results. According to Potron, this problem calls for a different organization of society, according to principles laid out by the Roman Catholic Church. Potron advocated that it is by a "corporative organization, supported in case of need by law […], that a supreme Arbitration Tribunal will be constituted, which will have the power to appraise and determine the expression of justice in exchange" (1912a: 314(92)). Here Potron quoted Desbuquois's (1911) article on the Law of the Just Price, where Desbuquois's proposal for a tribunal closely follows *Rerum Novarum*.[28] Potron commended this principle, but pointed out that the Tribunal's wisdom needs some assistance because complicated mathematical problems are involved: "This Tribunal, once it is constituted, will, by the very nature of things, be faced with problems that are the object of this study. It will thus be useful to attach a Bureau de Calculs to it" (1912a: 314(92)).

4.2. Collection and organization of the data

Potron's analysis is not limited to pure theory; it also reflects upon the practical role of the Bureau de Calculs. The activities must be based upon statistical data of production and consumption, collected and centralized by a competent statistical apparatus, and arranged in a vast and systematically structured tableau. In the notation of our section 3.1, this tableau takes the form $\begin{bmatrix} . & C \\ L & A \end{bmatrix}$. The m upper rows represent the "style of living" of the social groups, as described by the yearly consumption basket per member, and the n lower rows show the labour and commodity inputs necessary to produce goods, per unit of good produced. The m left columns provide information on the use of labour in the different industries, and the n right columns on the use of goods for domestic and industrial consumption. In Potron's numbering system the consumption goods are ranked first (he called these "units of the first kind" or "rations"), then the "mixed goods" (for example, Potron thought of coal, used both by households and industries), and then the pure capital goods (or "units of the second kind"). This hierarchy suggests that production adapts to consumption, but its special feature is that it reflects a practical procedure for the construction of the *tableau* in an industrial survey, as follows. As soon as the consumption goods are registered, the next step in the survey identifies their means of production, which are successively integrated in the *tableau* under construction, by adding columns (describing their use as inputs in the production processes of the already registered goods) and rows (describing their own production processes) (1937c: I, §3 (146-9)). In this way the *tableau* is expanding without disrupting the part already constructed. Potron depicted this *tableau* as a "sort of genealogical tree" (1942: 207(205-6)), whose size stops growing when all goods have been identified.

Potron emphasized that the relevant data can easily be found in the "the account-books of industries, businesses, housekeepers, housewives" (1912a: 290(70)). For the consumption

coefficients of the different social groups, Potron suggested that "accurate indications are given by the collective contracts" and by "the household budget" of a provident worker (1937c: I, §2 (146)). For the production coefficients, the numbers can be found in the same way in the industrial account-books. And Potron himself showed it: "a short survey in a bakery" enabled him to identify "the production coefficients for the formula of the cost price of a two kilogram loaf of bread" (1912a: 297-9(76-8)). Similarly, during a military term at the *Poudrerie du Bourget*, Potron needed only ten hours to compute the production coefficients for a sort of gunpowder called B-powder (1914a: 169-71; 1942). Instead of discussing Potron's treatment of fixed capital, overhead costs, insurance and administration, let us point out that the crucial difficulty Potron identified is that the industrial accounts are not always kept as scrupulously as in the military context of the *Poudrerie du Bourget.* Potron counted on the "collaboration of professional associations" and the pedagogical power of the survey, which he wanted to conduct in two steps: first a purely qualitative stage, creating just a catalogue of all sorts of goods and labour, then a quantitative one (1912a: 315-16(92-3)). Potron also thought he could rely on the self-interest of the producers, who would benefit from a better knowledge of their costs (see end of the 1942 paper). It is no surprise that Potron paid special attention to the quality of the industrial statistics in France, and so he wrote a letter to the *Journal de la Société de Statistique de Paris*, when he discovered that Alfred Sauvy's statistical programme did not specify how the labour time of each type of worker is distributed over each production process: "For my purposes, it is necessary that this distribution be specified" (1942: 208(206)).

4.3. The determination of a satisfactory regime: action or prayer?

Potron saw no spontaneous economic coordination: "Everybody chooses [his own production level] in a haphazard manner, and then one is quite surprised to have […] a large amount of employment or huge overproduction" (1935: 64(133)). The mission of the Bureau de Calculs

will be to determine a satisfactory regime of production and labour, and of prices and wages. To reach this objective, the tableau is a crucial tool.

The first problem is that of the very existence of a satisfactory regime. Taking into account the previous results and the equivalence between the relations (6) and (7), the existence question is as follows. Is the characteristic number *v* of the economic system smaller than the maximum number of working hours *N* or, equivalently, is the dominant eigenvalue of the matrix $M = A + LC/N$ smaller than one? Potron presented a criterion which does not require the computation of eigenvalues: it is necessary and sufficient that all principal minors of the matrix $I - M$ are positive (1913: 62-3(111)), or in the 1930s, equivalently and more efficiently, it is necessary and sufficient that all *leading* principal minors are positive (1937c: V; 1937d; 1939), a remarkable anticipation of the famous Hawkins-Simon condition.[29]

Suppose the condition is satisfied, thus ensuring the existence of satisfactory regimes of quantities and prices. How can they be computed? According to the duality result, the first step is to determine a satisfactory regime of production and labour. This defines the number of hours actually worked by each worker. Next, with respect to prices and wages, it is possible to eliminate the wages by combining inequalities (3) and (5), so as to reduce these to an inequality condition on prices only. In this procedure, each individual price inequality "can be constituted by a single producer, who has to take into account […] the standard of living *of his own workers only*. The determination of *effectively satisfactory* wages then does not cause any difficulty" (1937c: VI, §10 (196)).[30]

Can the Bureau de Calculs fulfil its mission in this way? In 1912, Potron underlined that the Bureau needs the support of the professional associations during the surveys, and even more when taking practical decisions "capable of inducing the progressive realization of an economic order in conformity with the general plan that God has drawn, but where He gives to men the task of carrying it out" (1912a: 316(93)). There is, however, a certain ambiguity in Potron's thinking on this matter. When talking about possible remedies against economic crisis,

he stated: “Computation cannot indicate the measures that should be taken to implement these changes. Here we meet questions where human freedom is at stake and which belong to the realm of pure economic science”. Yet, a few lines further, he also claimed: “Computation can *indicate* which modifications to the regimes of production and labour, prices and wage, are *certainly effective* to end the malaise” (1912a: 310(88)). The latter optimistic version dominates Potron’s discourse before the First World War. For instance, in an usually pedagogical publication, Potron (1914a) adopted a confident attitude towards the practical possibility of compiling all the coefficients of the entire French economy.[31] He estimated that there are about 10,000 different commodities, and that an average production process has about 200 nonzero input coefficients. So Potron claimed we need to compute about 2,000,000 nonzero commodity input coefficients, plus some sixty thousand labour coefficients. His mathematical background generated an optimistic judgement on the feasibility of his ambitions:

> The number of these constants is undoubtedly considerable; but a logarithmic table for ordinary use (an octavo volume of some five to six hundred pages) contains always at least 250,000 numbers of six to seven digits each. At most ten volumes of this type will therefore suffice to contain all the constants in question. With respect to the annual modifications which will be induced by the progress of the economy, a small booklet will be enough; as a matter of fact, one octavo page easily contains 500 numbers. The first determination represents therefore only the work required to calculate ten or so logarithmic tables. The updating work is insignificant. (1914a: 168)

In his later writings Potron became aware of the difficulties caused by the massive amount of numbers required and the statistical difficulties faced by the Bureau de Calculs. In his 1937 lecture series (1937c: VI, §7 (193)) Potron estimated that there are at least one million different “economic units”.[32] The complexity is effective as Potron did not aggregate the data: in his example of a unit of bread, he listed 73 inputs, including 1/1,000,000 of a clock and 1/500,000 of an inkstand (1912b: 248-9(98-100)). The amount of work involved in determining the characteristics of satisfactory regimes is “practically insurmountable, and it is only by tâtonnements that we can find a solution” (1937c: VI, §7 (193)). Things would have been easier “if one had started the calculations around a thousand years ago” (1935: 65(135)),

because at that moment the economic system involved much less equations. Potron remained convinced that updating the solutions every year would not be that big a problem, because the changes are small. The difficult issue is the determination of one initial satisfactory regime.

This explains Potron's interest in methods of approximation, when trying to determine a new satisfactory regime *par tâtonnements*, starting from a previous satisfactory regime. This interest is also related to Potron's remarks on the dynamics of population growth and technical progress. Potron stressed that the continuous growth of the population does not create computational problems (1913: 73-6(123-6)), as long as the proportions between different social categories remain the same. The discovery of a new production method might imply a significant change of some production or labour coefficients, and thus might disturb the satisfactory character of the old regime: "Technical progress, which should bring more wealth to the people for less labour, can often cause miseries and ruins" (1935: 64(133)). To avoid this, the Bureau de Calculs will have to replace, at the right moment, the current values of the variables by satisfactory values with respect to the new regime.

On the whole, in the 1930s, Potron switched to more pessimistic views on the possibility to manage the economic system. Mathematics can allow us to conceive the nature of the economic problems, but the solution seems to escape us: "The *only rational* procedure, that is, solving the systems […], is practically unrealizable, given the number of equations" (1935: 65(135)). Therefore, retreating from his early assertions, Potron concluded his 1937 lectures as follows: "This mathematical study, in the face of the economic problem, lets us feel the weakness of our human intelligence. It makes us repeat the traditional prayer to Our Father in Heaven: Give us this day our daily bread".[33]

5. CRITICAL REMARKS

We already drew attention to Potron's unusual treatment of prices and wages as purely arbitrary numbers which can be chosen freely. We now point out a few weak points of Potron's quantity system.

Let us first reconsider Potron's distinction between the principal unknowns (in the quantity system, this means the number of workers x_h of type *h* and the activity levels y_i) and the secondary unknowns, like the variables s_h, the number of nonworking consumers, which appear in our inequality (1). Potron emphasized that secondary unknowns can take on any positive value. An economic system, however, cannot support an arbitrarily high number of nonworking consumers, unless total production and thus the total number of workers is arbitrarily large. It is only in cases where Potron did not take total population as given, that he could treat the vector *s* of rentiers as a secondary variable. He was clearly aware that a satisfactory quantity system is defined by proportions between the variables, and that the size of the population is but a normalization factor. One could also defend Potron's distinction between principal and secondary unknowns by pretending it is of a purely mathematical order and constitutes a method for solving the set of equations, after which a factor of proportionality allows one to meet the total population constraint. But this pragmatic standpoint is not the one defended by Potron, when he stressed that the distinction is a crucial economic one (1935: 63, 64 (130, 133)).

On some occasions (for example 1936: first page (137); 1937c: II, §7 (156)), Potron did explicitly take total population as given. The determination of a satisfactory quantity system consists in computing the output levels, but also in partitioning adequately the population between the different categories of workers and rentiers (vectors *x* and *s*). In other words, when the system is fundamentally sound, Potron's solution of the economic malaise involves changes inside a social group (workers of type *h* become rentiers of type *h* or vice versa) or changes

from one group to another (professors become plumbers, by adopting their aptitudes and their way of life). Potron's answer to the unemployment problem assumes that his opinion on price adjustments ("modifications can be very fast", 1935: 63(129)) can be extended to different jobs and life habits, which is quite surprising in view of his repeated emphasis elsewhere on the importance of "real and concrete economic facts" (1911a: 7(61); 1912a: 300(79); etc.).

6. CONCLUSION

Potron is difficult to read, because of the gap between his texts and orthodox economics (unusual terminology and notations) and because of the mediocre quality of his exposition (accumulation of variables and equations, implicit or unrealistic assumptions). But once these difficulties are overcome, modern readers cannot but admire the originality and the audacity shown by Potron who, while embedding his reflection within the social doctrine of the Roman Catholic Church, tackled major economic problems (prices, wages, inflation, levels of output, allocation of labour, unemployment). Potron mobilized his mathematical talents to prove the existence of positive solutions in disaggregated linear models, by means of the Perron-Frobenius theorems, from 1911 on. He is the very first economist to state explicitly a duality property in these models.

Potron made a distinction between major crises which can only be solved by a change of the economico-social state, and economic malaises which correspond to bad proportions between the variables. To distinguish these two situations, Potron introduced the notion of the characteristic number of an economic system, and compared it with the maximum number of working hours per year. Because in general satisfactory solutions will not arise spontaneously, Potron advocated the creation of a Bureau de Calculs and outlined its tasks, including the construction of a *tableau*, a sort of comprehensive input-output table. Potron gradually became aware of the difficulties caused by the size of the problems (the number of goods and

equations, e.g.) and he looked for theoretical and practical methods of solution. His duality result provides a theoretical justification for using a hierarchical sequence in the problems which the Bureau de Calculs has to solve. Potron also presented different new tools of the Hawkins-Simon type when trying to check in practice whether solutions are possible.

In many ways Potron's modernity is surprising, as his approach to unemployment shows little realism, and as history has not created the economic, social or spiritual order that Potron wished to establish. Moreover, Potron's originality did not arise out of a conscious break with orthodoxy. As a matter of fact, Potron simply ignored the state of economic science of his times, and referred to scholastic ideas of just prices and wages, which were considered as long outdated by orthodox economists in the early 20th century. Unexpectedly, Potron in this way obtained important results that became part of mainstream economics only a few decades later.

Notes

[*] We have benefited from the generous assistance of the staff at the Municipal Archives in Vannes, the Military Archives in Vincennes, the National Archives in Paris, the Jesuit Archives in Vanves, and the archives of the Mouvement Chrétien des Cadres et Dirigeants in Paris, the Ecole Polytechnique in Palaiseau, the Université Catholique de Lille, the Université Catholique de l'Ouest in Angers and the Institut Catholique de Paris. We are grateful to Gilbert Abraham-Frois and Emeric Lendjel for various discussions, and to Robert Bonfils, Thomas Hawkins and Heinz Kurz for helpful comments. Chantal Dufour and Jocelyne Dufour have been successful in biographical researches. We are also grateful to members of the Salmon-Legagneur family, especially to Isabelle. A special word of thanks goes to Denise Potron (1908-2006) for her vivid memories of her uncle, Maurice Potron.

[1] This is explained at least partially by the disruption of the channels of communication during the war. When things returned to normal, Potron's death was mentioned in *Econometrica* (Oct. 1945, 13(4): 368). He had become a member of the Econometric Society in 1938 (*Econometrica*, Oct. 1938, 6(4): 397).

[2] The Potron bibliography by Abraham-Frois and Lendjel (2004: 209-10) contains 24 items. We have compiled a more extensive Potron bibliography with about 70 items in March 2007. The most recent version of our Potron Bibliography is available from any of the three authors on request.

[3] In his obituary, Barriol wrote "Clermont". It is highly probable that Barriol made a confusion with Clamart, where spiritual retreats were often organized by the Jesuits at the Villa Manrèse.

[4] Sources: 'Catalogus Universalis Nostrorum' (*Archives de la Province de France de la Compagnie de Jésus*, Vanves) and family records of Denise Potron.

[5] On Pupey-Girard, see du Passage's (1949) apologetic book. Desbuquois was head of the Action Populaire movement up from 1903 (Droulers, 1969, 1981). As noticed by Abraham-

Frois and Lendjel (2004: 31) in the interesting section of their preface devoted to social catholic reformers, Droulers is the only historian to mention Potron's economic works.

[6] The notation "(1912a: 314(92))" refers to page 314 of Potron's original 1912 text, which corresponds to page 92 of the Abraham-Frois and Lendjel edition. With regard to Potron (1937c) we give the number of the lecture (in Roman numerals) and the number of the paragraph, followed by the corresponding page of the Abraham-Frois and Lendjel edition. The bibliography of a later article (Potron, 1939) suggests that the text of the unpublished 1937 lectures was already available in "Paris, 1936, chez l'auteur".

[7] In an interview on the origins and the history of the USIC on the occasion of its 50th anniversary, Pupey-Girard (1943) mentions 515 members in 1907, 1,065 members in 1913, 6,400 in 1930 and almost 10,000 in 1940.

[8] See the forthcoming book on Frobenius by Thomas Hawkins (2008).

[9] The announced second part of the study was apparently never published.

[10] According to official documents of the institute (now in the *Archives de l'Université Catholique de l'Ouest*) Potron was in charge of a course on 'differential and integral calculus' in the period 1913-1919. His presence before the war might explain why a short unpublished and undated manuscript of Potron (1912c) is conserved in the library of the Catholic University of Angers. It is virtually impossible that he actually lectured in Angers in the period 1914-1918. The 'lending' of the title of doctor, with lectures effectively given by non-doctors, was a common practice in those days, even in peacetime. As a matter of fact, Potron sent a brief curriculum, on USIC stationary, to a correspondent in Angers and added: "If my name and my doctor diploma can, for want of a more active cooperation, be useful to you in some way, so much the better." (Letter to abbé Cisterne, dated 27 June 1912, *Archives de la Province de France de la Compagnie de Jésus*, Vanves).

[11] Source: interview with Denise Potron (December 2005).

[12] Source: file 'Maurice Potron' (*Archives de l'Armée de Terre, Service Historique de la Défense*, Château de Vincennes).

[13] A series of letters (now in the *Archives de l'Université Catholique de Lille*, Lille) confirms that Potron lectured there in the period 1921-1924.

[14] "Mr. Potron, in spite of a speech defect" ('Rapport sur la thèse de M. Potron', AJ[16] 5538, *Archives Nationales*, Paris); "Father Potron's stutter on some words is first surprising but soon becomes unnoticed" (letter of recommendation from Father R. Hollande to Mgr Baudrillart, 13 October [1924], *Archives de l'Institut Catholique de Paris*, Paris); "one of the Fathers Potron – he who stutters –" (letter probably of 1926 or 1927, *Archives de la Province de France de la Compagnie de Jésus*, Vanves); interview with Denise Potron; correspondence with former pupils of the Lycée Saint François-Xavier in Vannes.

[15] Both papers are available in the second edition of Le Chatelier's book (1934) on Taylorism.

[16] Cf. Potron's (1942) comments on Sauvy's paper (1941) on industrial statistics.

[17] This is the single use of the expression "supply and demand", and of "supply" in particular, in the work of Potron. An obvious difference between Frisch's and Potron's approaches is that Frisch (1934) did not refer to Perron-Frobenius results. Neither did Frisch ever refer to Potron.

[18] In his book on the contributions of French engineers to economics, Divisia (1951) did not mention Potron.

[19] Source: Vittrant's report in the file 'Maurice Potron' (*Archives de la Province de France de la Compagnie de Jésus*, Vanves).

[20] Abraham-Frois and Lendjel's initial formalization (2004, 2005) leaves no room for Potron's crucial constant N, the maximum number of workings days or working hours. The introduction of that magnitude in the latest version (2006) leads to a curious mixing of conditions (6) and (7), as the authors do not identify the matrices to which the Perron-Frobenius theorems apply (Bidard and Erreygers, 2007).

[21] All citations from Potron's work have been translated by the authors.

[22] The expression *état économico-social* sounds strange in French, but Potron never used the standard terminology *état socio-économique*. A "state" in Potron's sense describes the fundamentals of the economy. Potron makes use of the term *régime* when he refers to activity levels and the amounts of labour (*régime satisfaisant de production et travail*), or to prices and wages (*régime satisfaisant de prix et salaires*).

[23] Potron gave $N = 300$ days per year (1912a: 308(86)), $N = 313$ days, i.e. 365 days minus 52 Sundays (1913: 71(121)), and used $N = 3{,}000$ hours per year in the fictitious 'manna economy' (1937c: appendix, §4 (200)). No value of N is proposed in the six lectures of 1937. At the end of his letter to Gibrat (1935: 65(135)) Potron suggested that the 8 hours day diminishes the value of N but does not endanger the existence of satisfactory regimes. Holidays with pay were introduced in 1936 under the Front Populaire government in France and resulted in a significant reduction of the number of working days.

[24] The notations we use for vector inequalities are: $x \geq y$ means that the vector $x - y$ is nonnegative, i.e. its components are positive or zero; $x > y$ means that $x - y$ is positive, i.e. all its components are positive. Potron himself does not use vector notation.

[25] Following the tradition in mathematical economics, we use the expressions "Perron-Frobenius theorem" or "Frobenius theorem" for a wide set of results on eigenvalues of nonnegative matrices. For our paper the following results are important. Let the nonzero matrix M be real, square, nonnegative and indecomposable. Then its eigenvalue with largest modulus, called dominant root of M and denoted $dom(M)$, is real, positive and simple and is associated with a positive column (or row) eigenvector. For a given scalar α, there exists a positive column vector q (or a positive row vector z) satisfying the vector inequality $Mq \leq \alpha q$ (respectively, $zM \leq \alpha z$) if and only if $dom(M) \leq \alpha$. Many similar results on nonnegative

matrices are explicit or implicit in the classic article by Frobenius (1912). There is no trace of personal contacts between Potron and Frobenius, but Potron immediately saw he could apply the theorem to his economic model. Potron (1911b: 1129-30(63)) suggested that, in his extension of some Perron-Frobenius results on positive matrices to nonnegative matrices, he was inspired by a hint in an earlier article by Frobenius (1908: 475-6). See Potron's notes (1911a; 1911b; 1911c), where he already announced and summarized his forthcoming full articles (1912a; 1913). In this way, Potron already in 1911 mentioned some extensions, including the distinction between indecomposable and decomposable (*partiellement réduite*) matrices. Potron knew that eigenvectors of decomposable matrices may generate some zero quantities or prices. He also described a curious "semi-satisfactory regime" where a zero wage is however a just wage, because the corresponding workers need only consumption goods with zero prices (1913: 68-9(117-8), see also 1912a: 309(87)). Potron remarked that in case of decomposability it is possible to delete "some types of existence" and "results of labour" without affecting the remaining parts of production (1913: 69(118)).

[26] Extreme cases apart, no worker is out of labour in Potron's model (the non-workers, i.e. the rentiers, have a different status, even if they may switch to the category of workers). But it often occurs that the labourers work less than the maximum number N of hours or days: the idea of shared underemployment is closer to Potron's conception than that of unemployment (*chômage*).

[27] See also 1911a: 7(61); 1912a: 307(86); 1935: 65(134-5); 1936: last lines (139); 1937c: VI, §6 (193) and appendix, §7 (203).

[28] Compare the passage of Desbuquois's paper quoted by Potron (1912a: 91-2) and paragraph 78 of *Rerum Novarum*. We use the "authorized" English translation from 1891, which has 85 paragraphs and 40 footnotes, and is available on many websites. The original Latin version has no paragraph numbers at all. Many different versions and subdivisions exist. The "modernized"

English text on the Vatican website contains 64 paragraphs, where the content of number 58 is nearly equivalent to number 78 in the English translation from 1891.

[29] The economic literature attributes this criterion to Hawkins and Simon (1949). Many mathematical authors, for example Berman and Plemmons (1994, chapter 6), attribute the result to Alexander Ostrowski (1937/38). Potron's proof in 1937 relies on the LU-decomposition of a matrix, and he thanked Cornelius Lanczos for sending him a complete paper on this topic, extending a short published abstract (Lanczos, 1936). Actually, the Lanczos abstract describes another sort of matrix decomposition, the complete Lanczos paper has never been published and seems to be lost, and we have found no trace of correspondence between Potron and Lanczos. So the exact nature of the Potron-Lanczos connection remains a mystery.

[30] In modern (not Potron's) terminology: the construction of prices can be decentralized. But the argument holds only for an isolated producer who can treat the other prices as given. It does not hold for the determination of the whole price vector.

[31] This important paper has been overlooked by Abraham-Frois and Lendjel (2004).

[32] Not "10^8", as erroneously printed in the Abraham-Frois and Lendjel edition (p. 193). "Economic units" are goods, transports or "some less material things, [like] lessons of mathematics" (1937c: I, §1 (145)).

[33] We quote from Potron's handwritten text in the margin of a sheet of paper containing the programme of his six lectures at the Institut Catholique de Paris in 1937 (now in the *Archives de la Province de France de la Compagnie de Jésus*, Vanves). Here Potron also mentioned: "Applause at my final sentence".

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Mathematical Methods of Economics. Joel Franklin California Institute of Technology, Pasadena, California 91125. The American Mathematical Monthly, April 1983, Volume 90, Number 4, pp. 229â€“244. W. hen Dr. Golomb and Dr. Bergquist asked me to give a talk on economics, my first impulse was to try to get out of it.Â There can be no question, however, that prolonged commitment to mathematical exercises in economics can be damaging. It leads to the atrophy of judgment and intuition . . . . John Galbraith does not stand alone.Â If all of our nine voters have definite flavor preferences, the voters constitute 6 special-interest groups, corresponding to the six ways of ranking 3 flavors. For example, we might have the following tabulation: Individual values.