Fundamentals of

Electricity Derivatives

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Introduction

Deregulation of electricity is well under way in the United States and is starting in Europe. This represents a multi-billion spot market that is developing very quickly. And the same pattern of evolution as in the financial markets is being observed, with the growth of a variety of derivative instruments such as forward and Futures contracts, plain-vanilla and exotic options (Asian, barrier, etc.).

The main problem associated with the pricing of those derivatives is that the financial models do not capture the unique features of electricity, in particular the non-storability (except for hydro), the difficulties of transportation (access to network, disruption, etc.) translating into the non-validity of cash-carry arbitrage.

The goal of this paper is to investigate the possible approaches to the pricing of the most commonly traded electricity options .

Description of power options

The first category of options consists of calendar year and monthly physical options. The monthly options roughly follow the specifications of the electricity Futures contracts which were introduced on the New York Mercantile Exchange in March 1996. The exercise at the end of July of the August 1999 denominated call option allows the buyer to receive power (in a given location, defined in the option contract) during all business days (5 or 6 days a week, depending on the specification) of the month of August, 16 hours a day in most cases, from 6 a.m to 10 p.m prevailing time (on-peak hours), of a given number of megawatthours at the price k, the strike price of the option. Monthly options are fairly liquid and, as will be discussed below, relatively easy to hedge.

A second category of power options is comprised of daily options. These options are specified for a given period of time (year, season, particular month, etc) and can be exercised every day during this period. For example, the owner of the July-August 1999 daily call option can issue, if he so chooses, an advance notice on August 11 to receive a specified volume of electricity on August 12 during the on-peak hours, paying a price k per megawatthour. Daily options are not very liquid and are difficult to manage. (We note that although swing and other volumetric options also belong to this category, and daily options related issues discussed below are also relevant to these options, they raise additional constraints and complexities which are beyond the scope of this paper). Lastly, there are hourly options, designed to have access to power during specified blocks of hours (one, four, eight). As of now, the market for these options is thin.

In all three of the cases described above the option payoff at expiration is $\max(S_T - k, 0)$, where S_T is the spot price of electricity for the corresponding period of time.

There were several days last June when the spot price was above $2,000 per megawatthour, up from $25 a few weeks before (see Figs. 1, 2 and 3 for price and volatility data observed in ECAR - East Center Area Reliability Coordination Agreement - , region covering several Midwestern states). Sellers of calls, even deep out-of-the money calls such as k = $1,000, incurred severe losses. In the Spring of 1998, these options were selling for 50 cents per megawatthour, probably because $300 per megawatthour was the highest spot price of power registered during the year 1997.

These extreme spot prices coincided with the heat wave which had struck the Midwestern part of the United States, together with production and transmission problems. Generally speaking, power prices tend to be remarkably volatile under extreme weather conditions. Prices then become disconnected from the cost of production and may be driven very high by squeezes in the market due to generation shortages or transmission disruptions. Hence, power exhibits exceptional price risk, significantly higher than most other commodities like currencies, T-bonds, grains, metals, or even gas and oil.

Lastly, it is observed that financially settled power options are gaining popularity. The daily ones exhibit a 10 – 50% higher volatility than physically settled daily options. In order to not complicate issues further, we will restrict our attention in this paper to the physically settled options.

As mentioned earlier, the power market possesses some unique features :

a) Non-storability of electricity, and hence lack of inventories, requires the development of new approaches to study power markets, both from an economic and financial standpoint.

b) By necessity, US power markets (and this holds for Europe and other continents as well) are geographically distinct: there are several geographical regions between which moving power is either physically impossible or non-economical. This explains why new futures contracts are being created to cover these regions: after the COB (California Oregon Border) and PV (Palo Verde, Arizona) contracts introduced in 1996, the NYMEX recently started trading Cinergy contract (covering Midwestern region) and Entergy contract (Louisiana region). Another contract on PJM, whose delivery point is the border intersect of Pennsylvania, New Jersey and Maryland, has been recently introduced. Such geographical refinement of contracts is similar to the one observed in catastrophic insurance derivatives (see Geman 1994), first introduced in December 1993 by the Chicago Board of Trade for four regions, then extended to nine distinct regions in the United States.

c) The market for power options, like the credit market, is not really complete since hedging portfolios do not exist or are at least very difficult to identify, in particular for the daily options. This incompleteness implies the non-existence of a unique option price, hence the wide bid-ask spread observed on certain contracts.

In order to introduce a pricing methodology for power options, it is useful to first discuss the valuation methods used for other commodities, particularly energy commodities. In the next section we review the current approach to commodity option valuation, for both standard and Asian options (weather derivatives, which are becoming increasingly popular among power traders, most frequently have Asian-type payoffs).

Power versus Commodity Option Pricing

The notion of convenience yield was introduced by the economists Kaldor and Working who, among other topics, studied the theory of storage. In the context of commodities, the convenience yield captures the benefit from owning a commodity minus the cost of storage. Brennan and Schwartz in their pioneering research (1985) incorporated the convenience yield in the valuation of commodity derivatives and established in particular that the relationship prevailing between the spot price $S(t)$ and the future price $F(t,T)$ of a contract of maturity T is

$$F(t,T) = S(t)e^{(r-y)(T-t)}$$

where r, the risk-free rate, and y, the convenience yield attached to the commodity, are assumed to be non stochastic. This remarkable relationship allows one to interpret the convenience yield as a continuous dividend payment made to the owner of the commodity. Hence, under the additional assumption that the price of the underlying commodity is driven by a geometric Brownian motion, Merton's (1973) formula for options on dividend-paying stocks provides the price of a plain vanilla call option written on a commodity with price S, namely

$$C(t) = S(t)e^{-y(T-t)}N(d_1) - ke^{-r(T-t)}N(d_2)$$

where

$$d_1 = \frac{\ln\left(\frac{S(t)e^{-y(T-t)}}{ke^{-r(T-t)}}\right) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Note, that in the above situation market completeness does prevail, since we have only one source of uncertainty represented by the Brownian motion, and one risky asset, namely the underlying commodity, which can be sold, bought or stored to provide the hedging portfolio.

This implies the unicity of the price not only for plain-vanilla options but also for exotic options ; the latter case only involves solving mathematical technicalities . For instance, Asian options which represent today a huge percentage of the total number of options written on oil or oil spreads (because of the duration of oil extraction and transportation, most indices on oil are defined as arithmetic averages) are becoming popular in electricity, in particluar because of the Summer 1998 events . The averaging effect allows to smooth out the spikes in prices and keep the average cost of electricity over a given time period as the underlying source of risk in the option. It is well-known that the valuation of Asian options is a difficult problem and several approximations for the call price have been offered in the literature. Geman-Yor (1993) were able, using stochastic time changes and Bessel processes, to provide the Laplace transform of the exact price of the Asian option. Eydeland-Geman (1995) inverted this Laplace transform and showed the superiority of this approach over Monte Carlo simulations, in particular in terms of hedging accuracy. These results were established under the general assumptions of dividend payments for stocks or convenience yield for commodities.

As was mentioned before, the main difficulty in valuation of power options is due to the fact that electricity cannot be stored practically, which creates major obstacles for extending the notion of convenience yield to power:

a) By definition, the convenience yield is the difference between two quantities: the positive return from owning the commodity for delivery and the cost of storage. Because of the impossibility of storing power, these two quantities cannot be specified.

b) The non-storability of electricity also leads to the breakdown of the relationship which prevails at equilibrium between spot and future prices on stocks, equity indices, currencies, etc. The "no arbitrage" argument used to establish the cash and carry relation is not valid in the case of power, since it requires that the underlying instrument be bought at time t and held until the expiration of the futures contract.

c) There is another important consequence of non-storability: using the spot price evolution models for pricing power options is not very helpful, since hedges involving the underlying asset, i.e., the famous delta hedging, cannot be implemented, as they require buying and holding power for a certain period of time.

One way to avoid the problems described above and to extend to power derivatives the hedging strategy explicit in the Black-Scholes-Merton formula, is to use forward and future contracts. As we know from the analysis of these contracts in the case of stocks or equity indices, the dividend yield does not appear in the dynamics of forward and futures contracts (regardless of interest rates being deterministic or stochastic). Similarly, the dynamics of forward and futures contracts on commodities do not involve the convenience yield. Therefore, when these contracts are used to hedge power options (in particular, monthly or yearly options), the price of the option, which is by definition the price of the hedging portfolio, should not depend on the convenience yield. In other terms, even though we fully appreciate the economic interpretation of the convenience yield, we view it as embedded in more relevant state variables for the pricing of power derivatives.

Hence, for a given region and a given maturity T, we need to make an appropriate choice for the dynamics of power futures contracts $F(t,T)$. An example of how the futures prices depend on maturity T can be found on Fig. 4. Since volatility also varies with time t and maturity T, one has also to specify the forward volatility structure $v(t,T)$ which has, in the context of power, the property of increasing when t goes to T. (See Figs. 5, 6 and 7). In the next section we will discuss one approach to modeling evolution of the power forward curve.

A Production-Based Approach

We propose to approximate power future prices in the following manner

(1) $\qquad F(t,T) = p_0 + \varphi\,(w(t,T),\,L(t,T))$

where

- p_0 = base load price

- $w(t,T)$ = forward price of marginal fuel (gas, oil, etc.)

- $L(t,T)$ = expected load (or demand) for date T conditional on the information available at time t

- φ is a "power stack" function which can either be actual or implied from option prices.

If we assume that φ belongs to a two-parameter family of the type

(2) $\qquad \varphi = w\,\exp(aL + b),$

where a and b are positive constants, we obtain an exponential increase of the cost of generated power with increased demand, which is an adequate approximation to prices observed in the

power markets. Moreover, if we assume in a classical manner that the demand L is represented by a normal distribution, and that the forward fuel price is driven by a geometric Brownian motion, then from equation (1) we obtain that the quantity $F(t,T)$ (up to the constant p_0) is also driven by the geometric Brownian motion which has provided us with simple option pricing formulas for 25 years.

In reality, the power stack function may be more complex than the one proposed in equation (2). Figures 8 and 9 show that this function should be much steeper than the exponential one at the right end of the graph., where there is a quasi-vertical line for finite values of demand. In this region, a small change in load leads to a huge change in price and this will account for the spikes observed in practice. Moreover, the probability of higher values is in fact greater than in the log normal approximation we mentioned above and leads to the fat tails clearly exhibited by electricity price return distributions.

To summarize the above said, we note that in general to model the evolution of the power forward curve we need to model the evolution of fuel prices and demand, as follows from equation (1). However, under certain assumptions, such as (2), the evolution of $F(t,T)$ can be modeled using the standard Black-Scholes framework with an appropriately chosen volatility term structure. The hedges generated in this manner will be adequate to manage monthly and calendar year options. The situation is quite different for daily options.

The case of daily options

If the market of daily futures existed, the hedging of daily options would not be different from that of monthly options and the approach described above would be applicable. However, most markets – except perhaps for the Nordpool - do not have liquid daily forward or futures contracts, and therefore, we are forced to use an imperfect and sometimes dangerous surrogate of this daily futures contract in the form of the *balance of the month* contract. The balance of the month price is the price of power delivered every day from today until the end of the current month. Going one step further and assuming a strong correlation between this balance of the month and the spot price, we now can allow ourselves to model the spot price evolution in order to derive, in a standard way, the option price from the spot price dynamics. The balance of the month becomes the traded hedging instrument as opposed to the non storable spot. The main problems that one faces while modeling spot dynamics are the difficult issues of matching fat tails of marginal and conditional distributions and the spikes in spot prices. There are a number of techniques addressing these issues; below, we describe two models that appear to us most relevant.

i) The first one is a diffusion process with stochastic volatility, namely

$$
(3) \qquad
\begin{aligned}
dS_t &= \mu_1(t, S_t)dt + \sigma(t)S_t dW_t^1 \\
d\Sigma_t &= \mu_2(t, \Sigma_t)dt + y(t, \Sigma_t)dW_t^2
\end{aligned}
$$

where $\Sigma_t = [\sigma(t)]^2$, $W^1(t)$ and $W^2(t)$ are two Brownian motions, with a correlation coefficient $\rho(t)$, and the terms $\mu_1(t, S_t)$ and $\mu_2(t, \Sigma_t)$ may account for some mean reversion either in the spot prices or in the spot price volatility.

Stochastic volatility is certainly necessary if we want a diffusion representation to be compatible with the extreme spikes as well as the leptokurstosicity displayed by distribution of realized power prices. However, stochastic volatility puts us in a situation of incomplete markets since we only have one instrument, the spot power (or rather its surrogate) to hedge the option. Hence the valuation formula for the call,

$C(t) = E_Q \left[\max(S_T - k, 0) e^{-r(T-t)} \right]$, where r is the risk-free rate, S_T is the spot price at maturity as defined by equations (3), and Q is the risk-adjusted probability measure, would require the existence of a volatility-related instrument (for example, a liquid at-the-money option) that could be viewed as a *primitive* security and complete the market.

ii) A second model offers interesting features. As extreme temperatures, and hence, an extreme power demand, happen to coincide with outages in power generation and/or transmission, the spikes in electricity spot prices can be advantageously represented by incorporating jumps in the model (Geman and Yor, 1997, analyze an example of this type leading to completeness of the insurance derivatives market). A classical jump-diffusion model is the one proposed by Merton (1976)

$$dS_t = \mu S_t dt + \sigma S_t dW_t + U S_t dN_t$$

where

- μ and σ are constant $(\sigma > 0)$

- (W_t) is a Brownian motion representing the randomness in the diffusion part

- (N_t) is a Poisson process whose intensity λ characterizes the frequency of occurrence of the jumps

- U is a real-valued random variable, for instance normal, which represents the direction and magnitude of the jump.

This model has a number of interesting features. However, the assumption of risk-neutrality with respect to the jump component is totally lacking in credibility today (in the power derivatives market, for instance, the options described earlier now trade at 10 times the value at which they traded before the June 1998 spike). Hence, with one tradeable risky asset to hedge the sources of randomness represented by (W_t), (N_t) and its random multiplier U, we face an extreme situation of market incompleteness. In some recent popular models for credit derivatives and defaultable bonds, this incompleteness is even more severe since the intensity λ of the jump process is supposed to be stochastic. The latest world events demonstrate that this matter should be a serious concern.

Coming back to power derivatives, our view is that currently, the safest way to hedge daily power options is to own or lease a power plant. It is known that operating a merchant power plant is financially equivalent to owning a portfolio of daily options between electricity and fuel (spark spread options). Indeed, on any given day one should run a power plant only if the market price is higher than the cost of fuel plus variable operating costs. The net profit from this operating strategy is therefore :

$$\Pi = \max\left(\text{Price}_{\text{Power}} - \frac{\text{Heat rate}}{1,000}\,\text{Price}_{\text{fuel}} - \text{Variable costs},\ 0\right)$$

where Heat rate is a plant-dependent scaling constant introduced to express power and fuel prices in the same units. (Heat rate is defined as the amount of British thermal units needed to generate one kilowatt hour of electricity). The above expression is also the payout of the call option on the spread between power and fuel (spark spread), with variable costs being the strike of this call option. Owning the power plant is hence financially equivalent to owning a portfolio of spark spread options over the lifetime of the plant.

If the set of daily options we want to analyse matches this portfolio exactly, then its price should equal the value of the plant that may be obtained from economic fundamentals. (Note that this approach is the reverse of the one used in the theory of *real options*, introduced in corporate finance to value projects and investments. In practice, both viewpoints must be analyzed). Of course, in reality, an arbitrary portfolio of daily options will differ from the portfolio of spark spreads options representing the power plant, but the residual never explodes, even in the situation of extreme prices, hence can be hedged by classical techniques. For example, the difference between standard daily calls and calls on daily spark spreads in the case of high power prices depends only on the fuel prices, which have comparatively low volatility. Along the same lines, power arbitrage experiences by traders and marketers are not the ones we are used to in the financial markets. They consist in arbitraging the real options embedded in the business such as technology arbitrage : heat rates, fuel switching, response time ; transmission/transportation arbitrage or commodity arbitrage between gas, coal or hydro.

We addressed in this paper only some of the numerous issues related to modeling power prices, but probably some of the most important ones, particularly at the time when the intraday stock market volatility tends to resemble the power market volatility. In order to have a complete picture, we would need to incorporate the possible discontinuities due to power plants shutdowns, transmission congestion, changes in environmental policies (in particular regarding emission control) and development of new technologies to produce electricity.

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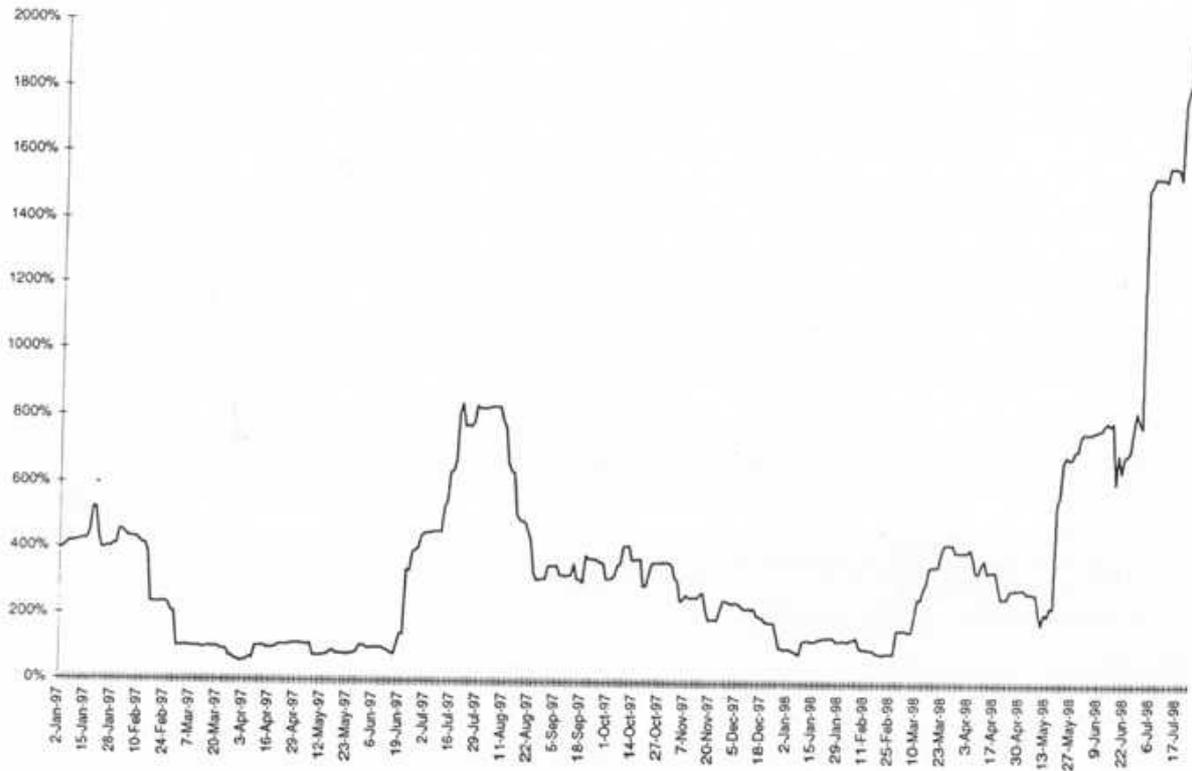
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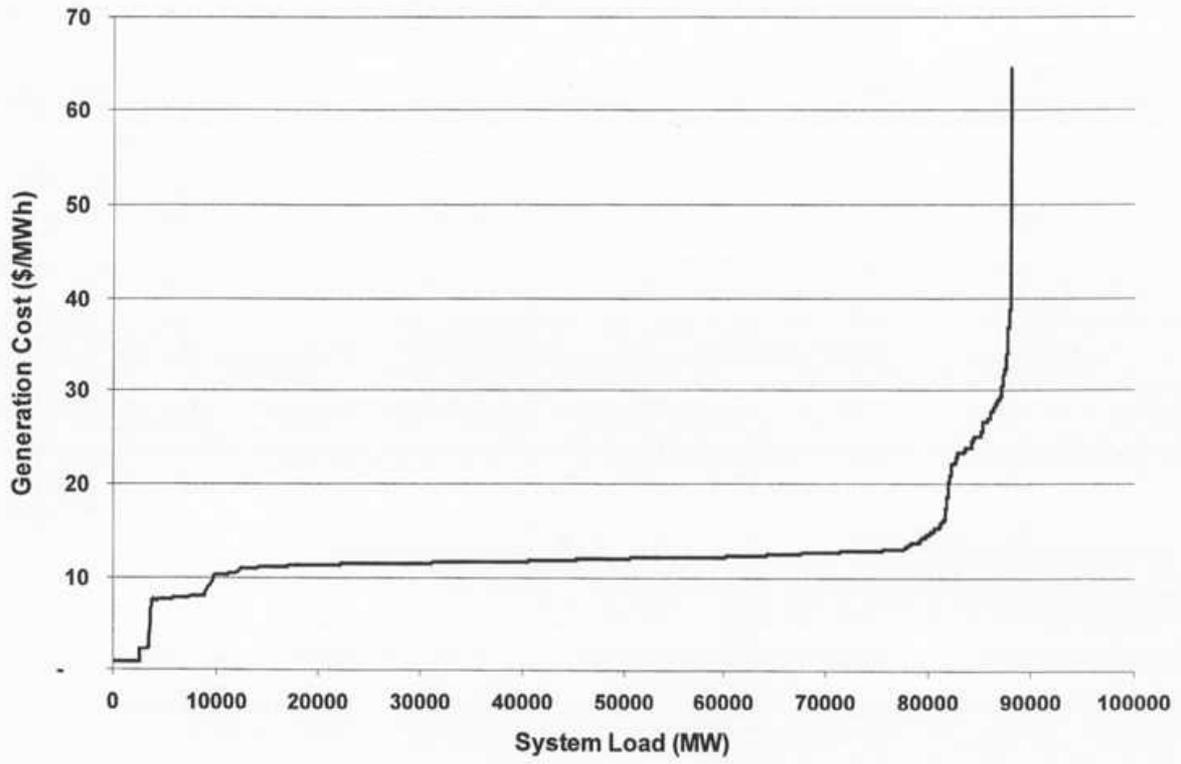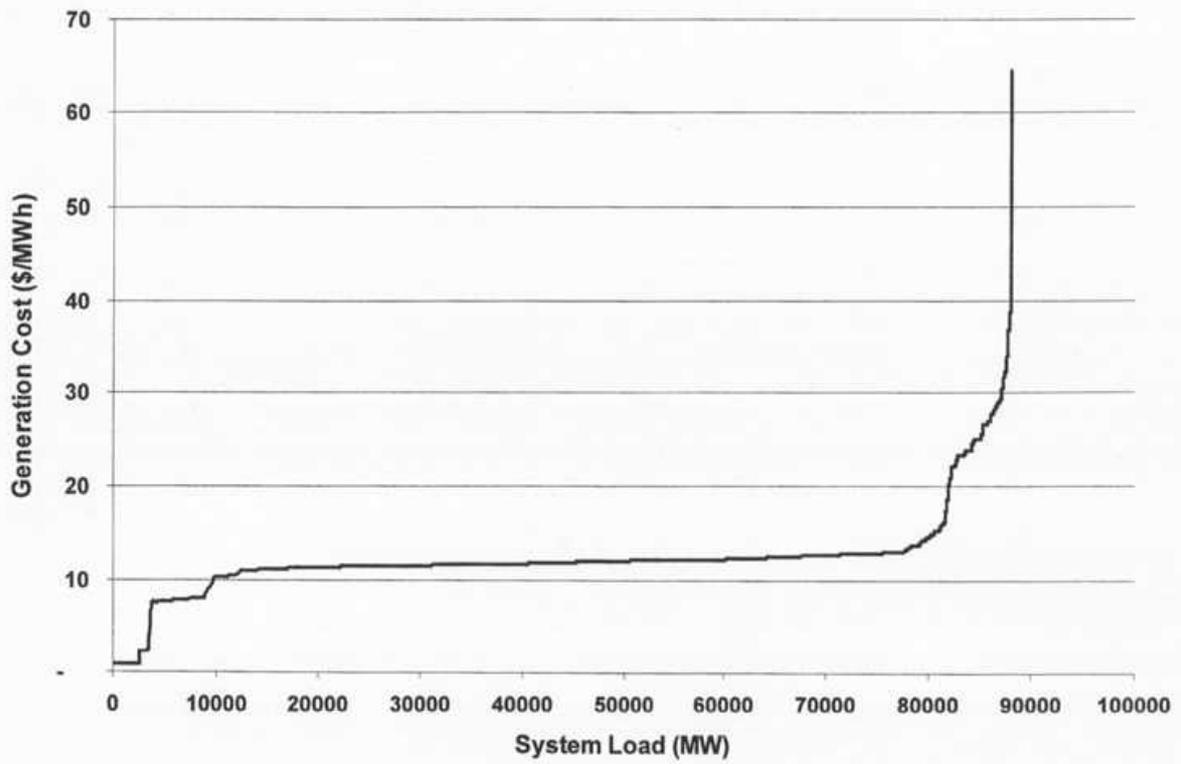
1. Price Levels (ECAR, On Peak, Firm)
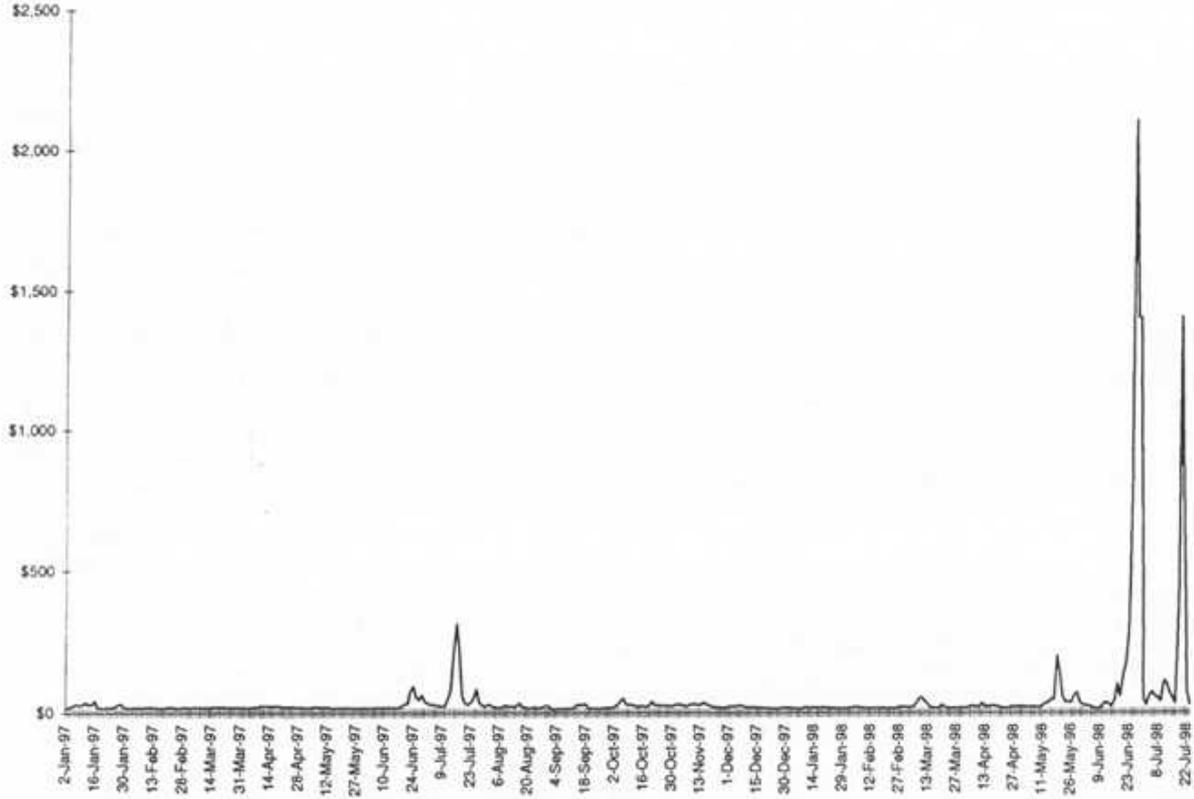
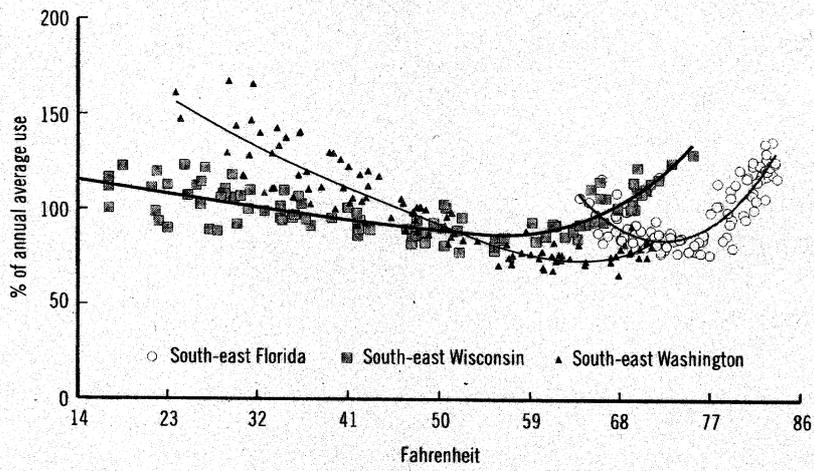3. Volatility (ECAR, On Peak, Firm, 20 Day Historical)

ɹ. ECAR GENERATION CURVE

ϑ. ECAR GENERATION CURVE
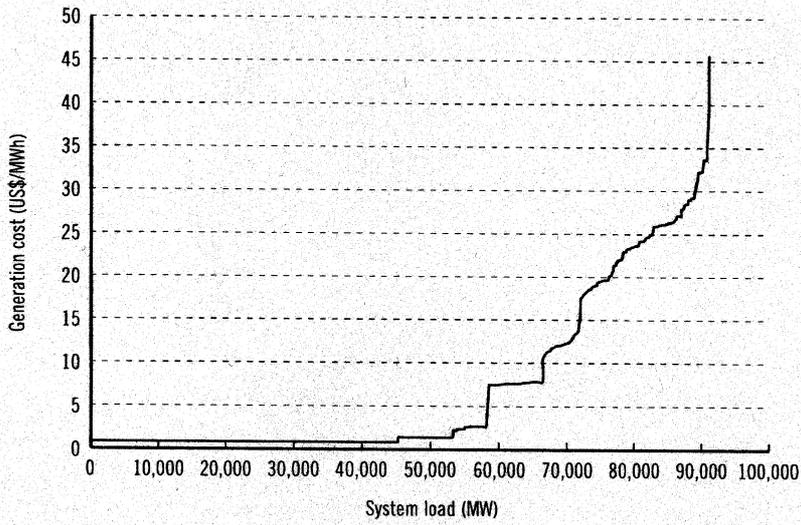
6. Residential electricity sales

Legend: ○ South-east Florida ▣ South-east Wisconsin ▲ South-east Washington

Y-axis: % of annual average use (0, 50, 100, 150, 200)
X-axis: Fahrenheit (14, 23, 32, 41, 50, 59, 68, 77, 86)

7. NPCC generation curve

Y-axis: Generation cost (US$/MWh) (0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50)
X-axis: System load (MW) (0, 10,000, 20,000, 30,000, 40,000, 50,000, 60,000, 70,000, 80,000, 90,000, 100,000)

8. ECAR generation curve

Y-axis: Generation cost (US$/MWh) (0, 10, 20, 30, 40, 50, 60, 70)
X-axis: System load (MW) (0, 10,000, 20,000, 30,000, 40,000, 50,000, 60,000, 70,000, 80,000, 90,000, 100,000)

20

We develop a fundamental model for electricity prices which lends itself well to integrating such forecasts, while retaining ease of implementation and tractability to allow for analytic derivatives pricing formulae. In an extensive futures pricing study, the pricing performance of our model is shown to further improve based on the inclusion of electricity demand and capacity forecasts, thus confirming the importance of forward-looking information for electricity pricing. Do you want to read the rest of this article? Fundamentals of Electricity Derivatives by Alexander Eydeland Southern Company Energy Marketing and Helyette Geman University Paris IX Dauphine and ESSEC. June 1999. Introduction Deregulation of electricity is well under way in the United States and is starting in Europe. This represents a multi-billion spot market that is developing very quickly. And the same pattern of evolution as in the financial markets is being observed, with the growth of a variety of derivative instruments such as forward and Futures contracts, plain-vanilla and exotic options (Asian, barrier, etc.).