THREE PHILOSOPHICAL PERSPECTIVES ON LOGIC AND PSYCHOLOGY: IMPLICATIONS FOR MATHEMATICS EDUCATION*

by

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It may be of use to distinguish and to relate to each other the logical and the psychological aspects of experience - the former standing for the subject matter in itself, the latter for it in relation to the child. John Dewey

Theoretically, it is important to ask what sort of correspondence exists between the structures described by logic and the actual thought processes studied by psychology. Jean Piaget

Abstract

Presuppositions regarding the proper relationship between logic and psychology are deeply embedded in any educational philosophy or theory regarding curriculum and instruction. Educational philosophers and curriculum theorists have long grappled with the relationship between the logical structure of subject matter knowledge and the psychological processes involved in understanding those structures. This paper considers educational implications of this fundamental relationship from analytic, pragmatic, and phenomenological perspectives. These perspectives are exemplified by their respective views towards mathematics and illustrated through their implications for mathematics education.

Introduction

What is 5 minus 8? What is 40 divided by 3? What is the square root of 2? What is the square root of -1? Even without a calculator, those who may recall their grade school mathematics, the answers to these questions are usually “figured out” in a manner based upon conventions and procedures that have been taught by rote and memorized by the learner. Answers to questions such as these are rarely understood, either by learners or by

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teachers, in any meaningful way. And yet these kinds of questions and the numerical domains which have been generated in addressing them—from whole numbers to negative numbers to rational numbers to real numbers to complex numbers—comprise the basic elements of mathematics itself.

In the realm of whole numbers there are no solutions to expressions such as "5-8"; nor within the domain of integers are there solutions to expressions "40÷3"; and so on. From a purely formal analytic perspective, the extension of numerical domains is logically implied by this lack of operational closure within pre-existing domains. On the other hand, these numerical extensions can also be seen as the result of a historical and psychological propensity of mathematicians towards systematization, completeness, and universality.

Mathematics is a befitting subject from which to study the relationship between logic and psychology. There are, and for at least the last twenty five hundred years there have been, profound and complex philosophical issues regarding the logical structure of mathematics in relation to the psychology of mathematical thinking. The ancient Pythagoreans posited numbers as the fundamental constituents of all things, be they objects of the senses or of the intellect. For Plato, mathematics afforded the natural course of passage into the realms of universal forms. He proposed mathematics as a centerpiece of his curriculum. The study of mathematics, he argued, provides a pedagogical bridge for learners to liberate themselves from the transient world of the senses and to regain entry to those transcendent realms.

Aside from its philosophical import and the study of mathematical structures *per se*, real world problems and applications have also motivated a significant amount of development in mathematics. Scientists, especially physicists, have long considered mathematics as the language in which the book of nature is written. To the extent that mathematics provides us with an intellectual understanding of the physical world—to the extent that mathematics provides a viable representation of reality—we remain, to this day, squarely within the age of Pythagoras. Thus, mathematics also serves as an important link in studying the relationship between philosophy and science. But this is not the relationship that will be of primary concern to us here. As Dilthey recognized some time ago, there are significant limitations in applying standard modes of scientific description and explanation geared toward our experience of the physical world to the inner psyche.

That realm of the psyche known as intellect is the realm in which logic has traditionally ruled. With the emergence of logic in ancient Greek thought, the natural affiliation of mathematics with physics and the Pythagorean unification of sense and intellect through the medium of mathematics, became more nuanced. Logic has served both as a means and justification for emancipating mathematics from intuition and the senses ever since the ways of truth and seeming were first revealed to and discerned by Parmenides. In the way of truth, the realm of intellect, things either are or are not. This strict bivalence established a logical basis for mathematical proof that at first complemented and then eventually replaced the use of intuitive graphical demonstrations.

The logic of mathematical proof has since provided the intellectual foundation for mathematics to the point of rejecting the intuitive representations that gave rise to the discipline in the first place. In the way of seeming, the realm of sense, things are blurred in that they can, simultaneously, both be and not be. Until recently, with the advent of fuzzy logic, there was no logic for this. Logic, or dialectic as it was referred to in the early days of its development, was concerned with identifying logical ways of reasoning and the laws of thought governing the way of truth. Eventually, rational thought itself was rejected as providing any basis for logic. Frege was adamant on this point in his rejection of any psychological ground to the discipline whatsoever when he noted that propositions can be thought, and propositions can be true, and we should never confuse these two things. Propositions that are thought are the concern of psychology, those that are true are the concern of logic.

To the extent that the logical structure of subject matter content remains at odds with the teaching and learning of those structures, educational theorists will be troubled by the relationship between logic and psychology. If logic and psychology are fundamentally different, as Frege insists they be, then should teaching conform to the logic of the subject matter, or should the subject matter conform to the psychology of the learner? Is it possible to do both? Is it possible to maintain the integrity of knowledge if logic is psychologized, as the pragmatic tradition would have it? Is it possible to maintain any sensitivity for the lived experience of the knower if the psychology of learning is logicized, as the analytic tradition would have it? If Frege is right, that we should keep these disciplines separated, and we don’t attempt to subsume one to the other, a better question might be: is there another way?

Husserl, a contemporary of Frege's, came to agree that it was important to keep logic and psychology separate. However, rather than simply abandon the study of one for the sake of the other, Husserl was motivated to develop phenomenology as an attempt to understand the relation between the two. In his early work on the philosophy of mathematics it is evident Husserl was struggling with the relation between psychology and logic, particularly with respect to mathematical understanding and the nature of mathematics. He was primarily concerned with how concepts such as number were grounded in and emerged from lived experience. This problem eventually revealed itself to be an exemplary case of the more general problem of phenomenology: the study of the objective structure of subjective experience.

Although much of Husserl's early work has been dismissed or ignored, there are grounds to suggest that phenomenology may offer a way of thinking about mathematics that can meaningfully bridge the gap between mathematics as a body of logically structured subject matter knowledge, and the psychology of mathematical thinking. If such a view can be exemplified and illustrated anywhere, it is in mathematics education.

Mind in Search of Method?

Exploring the relation between logic and psychology is complicated because there are so many variants of both. To lump these variants together, without some degree of qualification, would risk dealing in generalities that may hold little in the way of interest. However, any attempt to deal here with the manifold variations of logic and psychology would risk falling into irrelevance as well. Hopefully, a brief historical review of the main strands of thought giving rise to these two disciplines will suffice to provide some insight into the relation between them.

Psychology for Aristotle entailed the study of various qualities of the life force, from plants and animals to rational beings. Descartes' cleaving of the soul from the body placed philosophy in step with a long-standing theological distinction between the two. Perhaps as much a matter of jurisdiction as anything else, the material body, mechanized and deprived of spirit, came under the purview of science. Although the fate of the soul remained largely within the jurisdiction of theology, its psychology became a legitimate concern of rational and empirical philosophers alike.

Empiricists rejected the deductive logic underlying mathematics as the true method of natural philosophy. Bacon argued that the quest for scientific knowledge was better based upon the inductive method. Induction differs from deduction in that emphasis is placed upon generalized principles that are abstracted from, and contingent upon, particular observations. The success of empirical science, especially in physics, inspired philosophers such as Locke and Hume to attempt to adapt and apply the inductive method to traditional questions in epistemology and the philosophy of mind. With atomistic emphasis given to the foundational elements of sensation, along with causal principles governing their association, this strand of thought formed the basis of the emergence of psychology as an empirical science.

Rationalists such as Descartes, Leibniz, and Spinoza were convinced that the deductive logic underlying mathematics, especially Geometry, was the most appropriate method for addressing philosophical problems. Deduction, as characterized by Aristotle's Organum and exemplified in Euclid's Elements, involved positing self-evident truths and drawing logical implications from them. Aristotelian syllogisms constituted the deductive schemas for logical inference and were relatively uncontroversial. Despite the radical empirical implications of Descartes' *cogito*, there was much less agreement as to whether or not his criteria of clearness and distinctness were adequate for determining self-evident truths in either mathematics or philosophy—let alone for subsequent introspective methods of psychology.

The main problem of empiricism, revealed by Hume's critical assessment of his own attempt to fashion philosophy in the mold of science, was a problem with inductive reasoning itself. Hume realized that the inductive method could never determine anything with complete certainty and thus, having also rejected dogmatic rationalist appeals to self-evidence, he became skeptical regarding philosophy's aspirations for truth. Kant could not deny the force of Hume's critique. He was as unwilling to accept dogmatic rationalism as a foundation for philosophy as he was to accept Hume's skeptical empiricism as its outcome. Kant's response was notoriously complex, but the gist of it was to develop a new "transcendental" method of reasoning concerned with determining the necessary conditions for the possibility of human experience and understanding. Kant referred to these conditions as "synthetic a priori."

Prior to Kant it was standard practice for empiricists and rationalists alike to distinguish between various analytic judgments, propositions, or truths, from synthetic judgments, propositions, or truths. Judgments of the form “all X are Y” were taken to be analytic if the predicate was contained within the meaning of the subject and synthetic if it was not. It was generally accepted that analytic judgments were, one and all, a priori, or true independently of experience, and that synthetic judgments were a posteriori, or contingently dependent on experience. Kant, however, introduced a third possibility: the synthetic a priori—judgments that held true of any and every possible experience that could be known only through experience.

For example, Kant argued that our intuitions of objective experience, *qua* sensory experience of phenomenal objects, necessarily required space and time—for if objects lacked either spatial extension or temporal duration, objective experience (viz., experience of objects), would be impossible. Whereas the “forms” of space and time constituted necessary conditions of objective experience, Kant also argued that “categories” such as causality, quantity, etc., were necessary conditions for the possibility of understanding that experience. The forms and categories are manifested psychologically as schemas through which sensory experience and conceptual understanding are synthesized.

Thus, historically, three basic methods of reasoning and its application to pyschology emerged: the inductive logic of the empiricists, the deductive logic of the rationalists, and Kant’s transcendental logic. Kant’s insistence on the interdependency of sensory intuition and conceptual understanding eventually gave rise to the analytic-empirical method, an amalgamation of inductive principles with deductive inferencing, that is so prevalent in science today. Kant’s transcendental method of identifying necessary conditions for possible experience is viewed by many as a new form of rationalism.

Method in Search of Mind?

Today, the analytic-empirical approach of natural science and the conceptual analysis approach of analytic philosophy characterize two main orientations towards psychology. Although these methods are much more sophisticated and diverse than they were in the time of Bacon and Descartes, the empirical orientation of natural science in prioritizing the senses, and the rational orientation of philosophy in prioritizing the intellect, basically remains. For instance, the scientific disciplines of experimental psychology and psychophysics are clear applications of the analytic-empirical method. In contrast, analytic philosophies of mind and clinical psychologies in the Freudian tradition tend to rest more on rational than empirical foundations.

Despite universal recognition that observation and theory mutually inform each other, there remains a tendency to prioritize one over the other. From a logical perspective, the main issue seems to hinge upon whether one takes an inductive-empirical-observational approach or deductive-rational-theoretical approach to identifying first principles and justifying the grounds for their validity. Evidently the approach one takes towards identifying and justifying the assumptions with which one reasons will also serve to determine and justify one's approach to psychology. To the extent that logic is unconcerned with either the content or origins of the assumptions from which inferences are drawn, this issue is a meta-logical one.

On the other hand, the heart of the problem of understanding the relation between logic and psychology continues to hinge upon differences between sense and intellect: differences that have been pursued from a variety of psychological and philosophical perspectives. Thus, to the extent that the relation between sense and intellect is of concern to philosophy, this issue is a meta-psychological one as well, and the fundamental problem regarding the relation between logic and psychology may not be resolvable by either discipline. That is to say, this may be a problem that cannot be resolved by any attempt to subsume one to the other.

My intention here, however, is not to address this problem, but rather, only to give some indication of how deeply problematic the relation between logic and psychology is, and to suggest that it hinges in a deep and fundamental way on our understanding of the relation between sense and intellect. I will now turn to discuss problems that can result in mathematics education when either the logical structure of the subject matter or the psychological aspects of teaching and learning are prioritized over the other. I will then briefly illustrate an alternative phenomenological approach to mathematics education inspired by Husserl's early work in the philosophy of mathematics.

Analytic and Pragmatic approaches to Mathematics Education

Analytic approaches to education are primarily concerned with the logical structure of the subject matter and less concerned with the psychological factors involved in actually teaching and learning it. Such an approach is typically focused on the curriculum and how to present it—especially with respect to conceptual interdependencies between different subjects and with respect to the conceptual dependencies within each subject itself. The "new math" movement of the nineteen-sixties exemplifies the analytic approach to mathematics education. Indeed, this approach can be seen as a direct consequence of the formalist and logicist programs early in the twentieth century. Such an approach gives little, if any, consideration either to the historical origins or psychological content of mathematical concepts. A pedagogical manifestation of this analytic perspective is teaching and learning the logical structure of mathematics by rote and memorization with little, if any, appeal to intuition and real-world problems.

In contrast to analytic philosophy, pragmatism is more directly concerned with the lived experience of teachers and learners. Pragmatists tend to prioritize the empirical orientation of the empirical sciences over the predominantly rationalist orientation of analytic philosophy. Thus, pragmatic approaches to education are more naturally concerned with the psychological factors of teaching and learning. The constructivist movement that has followed upon the collapse of new math exemplifies the pragmatic approach in mathematics education. For some pragmatists, such as Piaget, logic serves less as a rational foundation for subject matter knowledge than it does as an operational model for psychological development. Constructivism, as a theory of learning, has been criticized for not focusing adequately on teaching. The constructivist response to this criticism, again in a pragmatic vein, has been to focus more on the functional utilitarian contexts in which mathematics is used rather than on the logical structure of the subject matter itself.

Today, constructivism is falling out of favor because of poor performances on national and international mathematics examinations. In part this may reflect a problem with implementing constructivist principles. It may also be a manifestation of a constructivist prioritization of psychology over logic. Whatever the case may be, disenchantment with constructivism has sparked yet another "back-to-basics" movement that appears, once again, to be focusing on the logical structure and formal procedures of mathematics at the expense of the psychological factors involved in teaching and learning the subject. Will the cycle be repeated? Could it be a cycle that is also a spiral—gradually converging on better understandings of the relation between curriculum and pedagogy, between logic and psychology, between intellect and sense? Where can we go from here?

A Phenomenological Approach to Mathematics Education

Analytic philosophy, with its orientation towards logic and conceptual analysis, is basically unconcerned with the subjective validity of lived experience. Pragmatism, with its orientation towards psychology and operationalism, is basically unconcerned with the objective validity of conceptual understanding. This is, of course, to emphasize extremes. Nevertheless, to forsake lived experience for conceptual analysis in mathematics education will inevitably be epitomized by phrases such as "math is what you do in math class." And when the logical structures and methods of mathematics are forsaken for the day to day applications of lived experience, learners are hampered from entering the pure conceptual realms Plato extolled so long ago. In cases where one perspective is not prioritized over the other, the end result often leaves it to the learner to puzzle over how the two may be related.

From his early logical and psychological investigations in philosophy of mathematics to his later phenomenological work in this area, Husserl was concerned with identifying and describing the origins of mathematical understanding in the phenomena of lived experience. As Farber reminds us, the original problem confronting Husserl was logical psychologism: the problem of whether logic can or should be considered in and of itself, independently of psychology. This, for Husserl, became a problem of reconciling the objective validity of logic and mathematics with the inherent subjectivity of lived experience. This problem may very well be considered as the central and defining problem of phenomenology, and of mathematics education for that matter.

Informed and inspired by Husserlian phenomenology, I have been attempting to envision what a phenomenological approach to teaching and learning mathematics might entail. The primary problem, of course, is determining the logical nature of mathematics in relation to the historical and psychological development of mathematical understanding in lived experience. This is not solely a matter of logical, psychological, or historical analysis, but requires a distinctively phenomenological method. I will not provide any detailed explication of my approach to phenomenological method here. Rather, I will simply provide a preliminary and abbreviated phenomenological analysis of perhaps the most basic and fundamental concept in the history of mathematics: the concept of an arithmetic unit.

Mathematicians often consider Euclid’s *Elements* as the definitive beginning of mathematics as a purely conceptual and logical endeavor. In book seven, Euclid’s definition of number is “a multitude of units,” with a unit being “that by virtue of which something can be considered one.” It is far from evident, however, what the phenomenological origin of the concept of an arithmetic unit is and what it actually means to consider something as “one.” As we shall see, it is not at all evident that the concept of an arithmetic unit is a singular concept at all. My basic approach has been to explore and reënact the kinds of questions being asked in pre-Euclidean Greek mathematical philosophy that gave rise to this concept up to the time of Euclid.

By the time of Pythagoras, pre-Socratic Greeks such as Thales and Anaximander, had already naturalized their mythological heritage by substituting physical elements such as water, air, earth, and fire, for roles traditionally occupied by the gods. Eventually, physical principles such as compression and rarefaction were added to the elements to account for how all things were generated and composed. It gradually dawned on the ancient Greek philosophers that these principles were not the usual kinds of things that were accessible to the senses, but had a purely conceptual or noetic quality about them. It is on this basis that Snell has credited the Greeks with the discovery of the intellect.

A fundamental philosophical problem, and possibly even the defining problem of philosophy, may have been to account for the unity of all things accessible to sense and intellect. Pythagoras—who according to Iamblichus purportedly coined the terms mathematics and philosophy—noted that the ratios of a monochord gave rise to phenomena harmonious to both sense and intellect. His solution to the problem of the unity was to propose a proto-atomic theory from which all things were composed of numerical units. These elementary units were physical in nature, in that they had spatial extension. All things were composed of these units and could be understood numerologically through the relations of the numbers of units by which they were composed. Some time later, Parmenides concluded that intellectually, in the way of truth, all things were actually one. The phenomenological thrust and import of this rather astonishing conclusion is that if something was truly of intellect in itself, it must be completely devoid of perceptual attributes.

relativize" universals to the particulars of lived experience via an intellectual process of separation, or abstraction. As Klein has brought out so well, this metaphysical shift in perspective enabled a shift in the *concept* of unit from one of discrete quantity to one of continuous measure. A unit of measure, in contrast to a unit of quantity, is divisible.

Here, within this brief synopsis of the origins of the concept of unit in ancient and classical Greek mathematical philosophy, lay the seeds for a rethinking of curriculum and pedagogy in mathematics education for the early grades. I would like to draw out a few educational implications from all this. First and foremost, it is helpful to remember that mathematics did not emerge, as we know this discipline today, from out of nothing. It has cultural and historical roots that provide clues as to its phenomenological nature and origins. Today, aside from the occasional historical vignette, the mathematics curriculum reflects the logical structure of what is being taught and pedagogy attempts to account for the psychological development of the learner. On the other hand, to simply recapitulate the historical development of these disciplines would be inappropriate as well. There is no need for psychological development to recapitulate historical development. Nevertheless, it is important to identify and describe necessary conditions for grounding the logical structure of mathematics in lived experience. Such is the task of phenomenology.

Secondly, the phenomenological approach I am taking in analyzing how elementary concepts such as the arithmetic unit emerge from lived experience places as much emphasis on identifying the questions that motivated these realizations as it does on the realizations themselves. In the case of the ancient Greeks, they were preoccupied with providing an account for the unity of all things. As Egan has so eloquently argued, these kinds of questions are by no means beyond the purview of children—quite the opposite, in fact. If anything, dealing with questions that involve generalities that are manifest in the lived experience of children are bound to be more accessible and of more interest to young children than arcane abstractions for which no grounding in lived experience has been given.

If the very general notion of an object is considered, one can find instantiations of this concept everywhere, not just in math class. By focusing, as did Plato, on the differences and similarities between various objects, one comes to see that there is no particular attribute that defines that concept. Comprehending the notion of an object that has no sensorimotor attributes whatsoever is the first step into the purely conceptual realms of mathematics. Moreover, children can learn quite readily to discern between objects that lose integrity when broken or divided from those that do not. Consider, for instance, that qualitatively, half a light bulb is no longer a light bulb, but half a cup of flour is still flour. Understanding the general concepts of divisible and indivisible objects of lived experience is an important phenomenological prerequisite to understanding conceptual distinctions in the arithmetic concept of unit which separate integer from rational numbers.

Finally, this much abbreviated phenomenological analysis reveals that the concept of a unit of quantity, upon which counting and whole number arithmetic is based, is fundamentally different from the concept of a unit of measure, upon which measuring and rational number arithmetic is based. Just as indivisible and divisible objects are very different kinds of objects, counting (in time) and measuring (in space) are very different kinds of activities. Perhaps it is not too surprising to find that these kinds of differences are reflected in the radically different metaphysical systems of Plato and Aristotle. Today, it is common practice in both curriculum and instruction to view whole number arithmetic as a “subset” of rational number arithmetic. This approach, typical of both modern logical and psychological perspectives underlying mathematics education today, leads to a conflation of fundamentally different concepts disguised under the same name, and different forms of arithmetic with different phenomenological foundations.

In conclusion, there is no claim being made here that a phenomenological perspective can solve the myriad educational problems that are associated with the recondite and abstruse relations between logic and psychology. However, insofar as it takes these relations as a central problematic, it is an appropriate and promising approach worthy of further consideration.

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This paper considers educational implications of this fundamental relationship from analytic, pragmatic, and phenomenological perspectives. These perspectives are exemplified by their respective views towards mathematics and illustrated through their implications for mathematics education.Â @MISC{Campbell_threephilosophical, author = {Stephen Campbell}, title = {Three Philosophical Perspectives on Logic and Psychology: Implications for Mathematics Education}, year = {} }.