Technical appendix of the book « Capital in the twenty-first century»

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http://piketty.pse.ens.fr/capital21c

In order to ease the reading of the book “Capital in the twenty-first century" and not to overwhelm the reader with footnotes, I have decided to include in this technical appendix the detailed presentation of historical sources, bibliographic references, statistical methods and mathematical models used in the book, including the presentation of the statistical series used in order to construct the figures and the tables presented in the different chapters.[1]

This appendix also includes additional figures and tables that are mentioned in the book (although not incorporated in it, so as to limit its volume), as well as Internet links to all the series, excel files, programs, formulas, primary sources and technical studies used as bedrock of this book.

Version available on line on July 18th 2014
The initial version of this appendix (February 1st 2014) is available here
An addendum to this appendix (chapter 10) was put on line on May 29th 2014

The page numbers referred to in this appendix relate to the edition of the book "Capital in the 21st century" published by Harvard University Press in March 2014.

[1] This technical appendix (as well as all figures, tables and data files) was translated from French into English by Pierre Bertrand. I am most grateful to Pierre for his excellent work.

Table of contents of the technical appendix

Text of the technical appendix: pp.1-85 of this pdf file

This technical appendix comes with a number of data files:

- on the one hand, excel files including for every chapter the set of figures, tables and statistical series, available in zip file format, or directly accessible in this directory

- on the other hand, detailed excel, stata and pdf files including raw data and intermediary calculations which the technical appendix of the book refers to, available in this directory.

It is also possible to directly access the figures and tables in the following directories:

- directory "figures and tables (pdf)" ou "(xps)"

- directory "supplementary figures and tables (pdf)" ou "(xps)"

The set of these files is gathered in this zip file.

Main technical references

"Capital in the twenty-first century" is based on fifteen years of research (1998–2013) devoted essentially to understanding the historical dynamics of wealth and income. Much of this research was done in collaboration with other scholars (see the acknowledgements in the beginning of the book). The main technical references and internet links to the related documents (on which most of the data presented in the book are based) are provided here. These works use a more technical style of writing compared to the book, which aims to offer a historical synthesis and gives few methodological details. On the contrary, the technical references given here provide all the required sources and methods, and will let the reader access to all the materials this book is based on. Additional references and documents are also available on my personal homepage.

T. Piketty, *Les hauts revenus en France au 20e siècle - Inégalités et redistributions 1901-1998*, Grasset, 2001, 807p.
[order the book] [technical appendix] [data series] [2]

T. Piketty, "Income Inequality in France, 1901-1998", *Journal of Political Economy*, 2003 [article in pdf]

T. Piketty, E. Saez, "Income Inequality in the United States, 1913-1998", *Quarterly Journal of Economics*, 2003 [article in pdf]

A. Banerjee, T. Piketty, "Top Indian Incomes, 1922-2000", *World Bank Economic Review*, 2005 [article in format]

T. Piketty, G. Postel-Vinay, J.L. Rosenthal, "Wealth Concentration in A Developing Economy: Paris and France, 1807-1994", American Economic Review, 2006
[article in format] [data files] [excel series]

[2] This book follows a first historical study (more limited in scope) that was published in 1998: See T. Piketty, "Les hauts revenus face aux modifications des taux marginaux supérieurs de l'impôt sur le revenu en France, 1970-1996", Cepremap, 1998, 179p. [article in pdf]. A condensed version of this study was also published in *Economie et prévision*, 1999 [article in format].

A. Atkinson, T. Piketty, *Top Incomes over the Twentieth Century - A Contrast Between Continental European and English-Speaking Countries*, Oxford University Press, 2007, 609p. [order this book]
(countries covered in this book: France, United-Kingdom, United States, Canada, Australia, New-Zealand, Germany, Netherlands, Switzerland, Ireland)

T. Piketty, N. Qian, "Income Inequality and Progressive Income Taxation in India and China, 1986-2015", *American Economic Journal: Applied Economics*, 2009
[article in format]

A. Atkinson, T. Piketty, *Top Incomes: A Global Perspective*, Oxford University Press, 2010, 776p. [order this book] [table of content (pdf)]
(countries covered in this book : India, China, Japan, Indonesia, Singapore, Argentina, Sweden, Finland, Norway, Spain, Portugal, Italy)

T. Piketty, *On the Long-Run Evolution of Inheritance: France 1820-2050*, Paris School of Economics, 2010, 424p.
[full document in pdf] [data series]
(short version: Quarterly Journal of Economics, 2011) [article in pdf]

A. Atkinson, T. Piketty, E. Saez, "Top Incomes in the Long Run of History", Journal of Economic Literature, 2011 [article in pdf]

F. Alvaredo, A. Atkison, T. Piketty, E, Saez, World Top Incomes Database (WTID), 2011-2013 [WTID database]
(database including updates for the set of countries)

T. Piketty, G. Postel-Vinay, J.L. Rosenthal, "Inherited vs. Self-Made Wealth: Theory and Evidence from a Rentier Society (Paris 1872-1937)", *Explorations in Economic History*, 2014 [article in pdf] [working paper in pdf] [data series]

T. Piketty, G. Zucman, *Capital is Back: Wealth-Income Ratios in Rich Countries, 1700-2010*, Paris School of Economics, 2013, 976p.
[full document in pdf] [data series]
(short version : article in pdf , Quarterly Journal of Economics, 2014)
(covered countries: The United States, Japan, Germany, France, the United-Kingdom, Italy, Canada, Australia, Spain)

T. Piketty, G. Zucman, "Wealth and Inheritance in the Long Run", Handbook of Income Distribution, vol.2, edited by A.B. Atkinson and F. Bourguignon, North Holland, 2015.
[article in pdf]

« Capital in the twenty-first century » is also based on some theoretical research dedicated to optimal taxation and in particular the following articles:

T. Piketty, E. Saez, S. Stantcheva, "Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities", *American Economic Journal: Economic Policy*, 2014
[article in pdf] [data files]

T. Piketty, E. Saez, "A Theory of Optimal Inheritance Taxation", *Econometrica*, 2013
[article in pdf] [data files]
Version longue: "A Theory of Optimal Capital Taxation", NBER WP 2012
[article in pdf]

T. Piketty, E. Saez, "Optimal Labor Income Taxation", *Handbook of Public Economics*, volume 5, chapitre 7, 2013, North-Holland.
[article in pdf]

Appendix to the introduction

Figure I.1. Income inequality in the United States, 1910–2010 (**p.24**)

The series used to construct Figure I.1 (top decile share in United States total income from 1910 to 2010) replicated in the book on p.24, are available on Table SI.1, as well as in the corresponding excel file.

Table SI.1. The top decile income share in the United States, 1910
(*series used for Figure I.1)*

These series were initially produced and published in « Income Inequality in the United States, 1913-1998 » (article *QJE* 2003, co-written with E. Saez, available here). They were constructed on the basis of income tax return data (used so as to estimate the top decile of income) and national accounts (used to estimate total income of the population as a whole), following similar methods to those applied in the case of France in *Les hauts revenus en France au 20e siècle*…, Grasset 2001 (see above for the main technical references used in the research and the corresponding internet links). This method is detailed in these two publications and rest on the extension of the method used for the first time by Kuznets in his book written in 1953. [3]

All the series are updated and published every year as part of the "World Top Incomes Database" (WTID). They are analyzed in a detailed manner in the third part of the book (see in particular chapters 7, 8, and 9)

The series indicated in table SI.1 (and used for figure I.1) corresponds to series which are available in the WTID at the date of writing of the book (first semester of 2013). Updated series will be available in the coming months and years on the WTID website, which we recommend to the readers that are interested. Preliminary data available for 2011-2013 suggest that the increase of the top decile share in US total income is still going on.

It must be notified that the series hereby indicated correspond to the series including the realized capital gains, in the nominator (top decile income) as well as in the denominator (total income of the whole population). The comparison with the series

[3] See in particular *Les hauts revenus en France au 20e siècle…*, appendix A and B for a very precise description of this method, allowing everyone to duplicate the whole set of calculations, from the fiscal sources to the final series.

excluding capital gains is made in chapter 8 (see in particular figure 8.5 and table 8.2)

Finally, the series indicated were extended back to 1910 by drawing on the estimates obtained from the declarations of income as of 1913 (first year of enforcement of federal income tax) as well as estimates of the American income distribution established before World War I.[4] Details are given in the excel files of the introduction and of chapter 8.

Figure I.2. The capital/income ratio in Europe, 1870–2010 (**p.26**)

The series used to construct Figure I.2 (the capital/income ratio in Europe, 1870-2010, calculated with decennial average) replicated in the book on p.26, are available on Table SI.1, as well as in the corresponding excel file.

Table SI.2. The capital/income ratio in Europe, 1870-2010
(Series used for figure I.2)

These series have been taken from *Capital is Back : Wealth-Income Ratios in Rich Countries, 1700-2010* (Paris School of Economics, 2013, written with G. Zucman, available here). These were established with the collection and the harmonization a huge number of national accounts as well as country balance sheet produced in several countries since the early 18th century. All the details on the sources and methods used can be found here. These series are analyzed in a detailed manner in the second part of the book (see in particular chapters 3, 4 and 5).

It must be specified that the series indicated in table SI.2 (and used for figure I.2) refer to the ratio between private capital (the total of financial and non-financial assets held by households and non-profit institutions serving households, net of financial liabilities) and the national income. Similar series referring to national capital (sum of private and public capital) in Europe are analyzed in Chapter 4 (see in particular figure 4.4-4.5).

Also, the series indicated in table S.I.2 and the figure I.2 correspond to decennial average (not to annual series, which differ from the series indicated in table S.I.1 and

[4] WTID series for the United States start in 1913 for the top centile and in 1917 for the top decile. In the first years of its enforcement, the income tax only applied to a small fraction of the population, and therefore do not indicate the total income of top decile. This phenomenon can be observed in other countries.

figure I.1). The benefit of decennial series lies in the fact that it is possible to go beyond the short-term variations in order to focus solely on the long-term evolutions. The decennial averages presented in the book usually correspond to averages established over the years 1990-1999 (for the "1990" value), 2000-2009 (for the "2000" value) and so forth, with several exceptions due to particularly abrupt breaks which make the derivation of a complete decennial average meaningless (for instance the "1910" value corresponds to the average of the years "1910-1913"). With regards to older periods of time, in particular for the 18th century and the first half of the 19th century, there are no annual series available, and the decennial series correspond to isolated estimates. For the 2000 decade, only the estimations of the wealth evaluated as of January 1st, 2010 and January 1st, 2011 were available for most countries when the book was written (first semester of 2013), and the "2010" value simply corresponds to the average of these two estimations. Preliminary data available for 2011-2013 suggest that the levels indicated for "2010" for the different countries are approximately correct. All annual series (and all the corresponding annual figures) and updates are available here. [5]

[5] The series regarding the historical evolution of the ratio presented in the book correspond to series available in the Piketty-Zucman database on the date of writing (first semester of 2013). These series are regularly updated and revised, and the readers interested in the latest versions are invited to consult this database (which should soon be integrated in the WTID to form the "World Wealth and Income Database" (WWID)).

Appendix to Chapter 1: Income and Output

The notion of national income: capital depreciation (**p.43-44**)

I indicate on p.43-44 that capital depreciation usually represents about 10% of the GDP. More specifically, the capital depreciation is of the order of 5-10% of the GDP in the least advanced economies (it is typically just about 5% of the GDP in traditional agricultural economies where the capital mainly takes the form of land), and can rise up to 10-15% in the most developed economies (where an important share of the capital is comprised of elements that depreciate more rapidly, such as computer equipment). For detailed tables on the historical evolution of the capital depreciation in different countries, see *Capital is Back...*, 2013, Table US.8, JP.8, etc.

More generally, for detailed tables displaying the breakdown from GDP to national income and indicating different decompositions of the national income and their historical evolution in different countries, see *Capital is Back...*, 2013 (in particular Tables US.8-13, JP.8-13, etc. et A40-50).[6]

The definitions used hereby are the internationally standardized norms of national accounting that are currently in force under the aegis of the United Nations and the European and International institutions. For a presentation of these norms and their recent evolution, see *Capital is Back...*, 2013, Annex A.

The notion of national income: figures on net foreign income (p.45)

I specify in p.45 of the book that the net foreign income currently represents just about 1-2% of the domestic output in most of the wealthy country. For detailed tables indicating the evolution of the ratio (national income)/(domestic output) in wealthy countries, see *Capital is Back...*, 2013 (see in particular Table A40 et Figure A57). For data on all the countries of the world, see for instance the series gathered in this table extracted from national standardized accounts collected by the United Nations for the recent decades.

[6] For detailed tables related more specifically to France, see also *On the Long-Run Evolution of Inheritance...*, 2010, Annex A, as well as appendix tables presented on the website http://www.revolution-fiscale.fr (see in particular table CN1-23).

Examples of ratio capital/income (p.50-51)

I indicate on p.50-51 approximate orders of magnitude for the capital/income ratios in force in the wealthy countries in the early 2010s. For the exact numbers, extracted from officials national accounts, see *Capital is Back...*, 2013, Tables US.1, JP.1, etc.

As regards France in 2010, for example, the national income is 1701,7 billion of euros (which represents an income of 34 962 euros per capita on average), net private wealth is 9 777,2 billions of euros (which represents an average wealth of 200 878 euros), hence a ratio wealth/income β=575% (see Table FR.1).

Detailed references on historical national accounts (p.57)

A detailed bibliography on historical national accounting, in particular regarding the first national accounts considering wealth and income established in the United Kingdom and France since the early 18th century, is provided in *Capital is Back...*, 2013 (see in particular annexes by countries).

Figure 1.1. The distribution of world output, 1700-2012
Figure 1.2. The distribution of world population, 1700-2012
Figure 1.3. Global inequality 1700–2012: divergence then convergence?
Table 1.1. Distribution of the world GDP, 2012
Figure S1.1. The distribution of world output, 0-2012
Figure S1.2. The distribution of world population, 1700-2012
Figure S1.3. Global inequality 1700-2012: divergence then convergence?

The series used to construct Figure 1.1, 1.2, 1.3 and Table 1.1, replicated on p.60-63 of the book, as well as supplementary figures S1.1, S1.2 and S1.3 are available on Table SI.1, S1.2 and S1.3 as well as in the corresponding excel file. The supplementary tables S1.4, S1.5 and S1.6 that offer different variants of Table 1.1, are derived from the same data.

Table S1.1. The distribution of world output, 0-2012
Table S1.2. The distribution of world population, 0-2012
Table S1.3. Per capita GDP, 0-2012
(series used for Figures 1.1- 1.3 et S1.1-S1.3)

To create these summary tables on the very long-term evolution of the world's GDP distribution and world population, I principally used the historical series of Angus Maddison, "Historical statistics of the world economy 1-2008" (February 2010) for the period 0-1990, which I completed with officials series from the United Nations/World Bank (October 2012) for the period 1990-2012. Russia was included in Europe, and the former republics of central Asia as well as Oceania were included in Asia. All the details are provided in the formulas and the excel file to which the formulas refer to, in particular in the following files (which contain detailed data for the countries):

Chapter1TablesFigures.xlsx
MaddisonWorldGDPSeries1to2008.xls
UNPopulationSeries19502100.xls
WBWorldGDP.xls

The following point must be clarified. With regard to the population, Maddison's historical series are perfectly consistent with the official series (no homogenization is required). With regard to GDP, however, I had to slightly modified Maddison's series in order to harmonize them with the official series for the recent period. The main difficulty lies in the fact that Maddison's series on the recent period indicate a Chinese GDP substantially higher than official series. This does not mean that Maddison's evaluation could not be more accurate, or that future ICP studies (international studies designed to estimate purchasing power parities among countries) will not conclude that Chinese GDP must be raised. Nonetheless, in the current situation, the results of the 2005 ICP study are widely accepted by international institutions involved (World Bank, Eurostat, OECD, IMF, UN, etc.) [7], and form the base of comparison commonly recognized.

Therefore I decided to modify Maddison's series on this basis so as to ensure consistency with the official series. For a detailed analysis of this controversy, and a description of the corrections provided, see excel file (CorrectionsMaddison paper and CorrectedSummaryTables). See also, the dissenting point of view of Maddison

[7] National accounts worldwide published by the different institutions over the recent period are almost identical. The most complete decompositions for the largest number of countries are the ones published by the United Nations, however the most complete PPP series are the ones published by the World Bank (The UN does not publish PPP series, but publishes more detailed decompositions of national accounts by country, including on *factor shares*).

and Deaton, and the detailed results of the 2005 ICP study.[8] The final results of the 2011 ICP study are expected for 2014-2015.

Note on measures of world income inequalities (p.59-63)

Figure 1.3, which indicates Europe-America and Asia-Africa continents position vis-à-vis average global GDP, provides an elementary measure of the output and income global inequality. By combining Maddison's series on the evolution of the global GDP distribution amongst the countries with the estimations of incomes inequality inside the countries, it is possible to develop more sophisticated measures. As indicated on p.59 of the book, pioneer works on those questions were carried out by François Bourguignon and Branko Milanovic. See especially the following papers: Bourguignon-Morrisson 2002, Milanovic 2009, Sala-i-Martin 2006.

Overall, the conclusions obtained are close to those indicated on figure 1.3: strong increase of global incomes inequality during the 19th century and the largest part of the 20th century (as the gap between non-industrialized and industrialized countries is growing), then stabilization and/or diminution as of the last decades of the 20th century and early 21th century (considering the very strong growth registered in emerging countries).

Figure 1.4. Exchange rate and purchasing power parity: euro/dollar
Figure 1.5. Exchange rate and purchasing power parity: euro/yuan
Figure S1.4a. Exchange rate and purchasing power parity: euro/rupee
Figure S1.4b. Exchange rate and purchasing power parity: euro/yen
Figure S1.5a. Exchange rate and purchasing power parity: dollar/yuan
Figure S1.5b. Exchange rate and purchasing power parity: dollar/rupee
Figure S1.5c. Exchange rate and purchasing power parity: dollar/yen

The series used to construct Figure 1.4 and 1.5, replicated on p.65-66 of the book, as well as supplementary figures S1.4-S1.5 are available on Table SI.7 as well as in the corresponding excel file.

[8] The two principal publications from 2005 ICP study are the following: *Global Purchasing Power Parities and Real Expenditures - 2005 International Comparison Program* (World Bank, 2008); *Measuring the Real Size of the World Economy - The Framework, Methodology, and Results of the International Comparison Programme* (World Bank, 2011). See for example here and there.

Table S1.7. Exchange rate and purchasing power parity, 1990-2012
(series used for Figures 1.4-1.5 et S1.4-S1.5)

The series indicated in this table correspond to the official series (World Bank/OECD/IMF) of annual exchange rates averages and of purchasing power parity from the latest available international studies (ICP 2005). As explained above, it is possible that future ICP studies on purchasing power parity result in substantial modifications.

Note on the different prices among the countries (p.65)

I note on p.65 that in the latest available study (ICP 2005), certain prices are indeed higher in Europe (such as energy, housing, hotels and restaurants), while other are substantially lower (such as health and education). See *Global Purchasing Power Parities and Real Expenditures - 2005 International Comparison Programme* (World Bank, 2008), Table 2, p.38-47. It must be noted that in the national accounts the price of public services provided for free - or for a reduced price – is measured by their production cost (for example the teachers' salary for education), which is paid ultimately by the taxpayers. It is an imperfect statistical convention, but it is a lot more satisfying than not taking them into account, which would completely distort international comparison (for a deeper discussion on this subject, see chapter 2).

Comparisons between incomes and output inequality (p.67-68)

Comparisons made in p.67-68 of the book between income and output global inequality are based on the following information. For detailed tables indicating the evolution of the ratio (national income)/(domestic output) in wealthy countries, see *Capital is Back...*, 2013 (see in particular Table A40 and Figure A57). For data regarding all the countries, see for instance the series collected in this table from standardized national accounts gathered by the United Nations for the recent decades. For Africa, the ratio does not exceed 90% during the last decades (just about 90% for numerous sub-Saharan countries in Africa). For data allowing a comparison between capital income flows and international aid flows, see this table. Will consider the following points. At the global level, total aid flows represent about 0.2% of the global GDP, and appears stable since the 1970s (the official objective is 0.7% of the GDP in the wealthy countries) The aid received is approximately 1% of the GDP in Africa in PPP and 2% at the current exchange rate (in Sub-Saharan Africa: 2% and 4%). In all cases, aid inflows are below aid outflows. Moreover, official

aid flows include technical support (which is for a large part salaries for consultants and experts from the wealthy countries) and its measure is highly controversial. For data on the magnitude of the wealth position of rich countries vis-à-vis the rest of the world at the colonial time, see chapter 3 and 4.

Study on international trade gains (p.71)

I refer in p.71 of the book to a study on the gains brought by the opening to international trade. This is the Levchenko-Zhang 2012 paper. On this subject, see also Costinot-Rodriguez 2013.

Appendix to chapter 2: Growth: myths and realities

Figure 2.1. The growth of the world population, 1700-2012
Figure 2.2. The growth rate of the world population from Antiquity to 2100
Figure 2.3. The growth rate of per capita output since the Industrial Revolution, 1700-2012
Figure 2.4. The growth rate of world per capita output from Antiquity to 2100
Figure 2.5. The growth rate of world output from Antiquity to 2100
Table 2.1. World growth since the Industrial Revolution
Table 2.3. Demographic growth since the Industrial Revolution
Table 2.5. Per capita output growth since the Industrial Revolution

The series used to construct Figure 2.1-2.5 and tables 2.1, 2.3 and 2.5, replicated on p.75-101 of the book, are available on Table SI.1-S1.3 (chapter 1) and S2.1-S2.4, as well as in the corresponding excel files.

Table S2.1. World growth from the Antiquity (growth rate per period)
Table S2.2. Growth rate of world population, 0-2100
Table S2.3. Per capita output growth since the Industrial Revolution
Table S2.4. World output growth rate, 0-2100
(*series used for Figures 2.2-2.5)*

To construct these summary tables on the evolution over a very long period of global growth, I used the same sources than the ones indicated in Chapter 1 (Maddison's historical series for the period 0-1990 and the UN/WorldBank series for the period 1990-2012). Estimates regarding population growth for the 21st century are supposed to evolve in accordance with the United Nations median scenario, and are explained in chapter 2 for what regards growth per capita. All the details are provided in the related excel files, and in particular in the chapter 2 excel file.

Issues related to the calculation of the public sector's production (p.92)

The best way to illustrate the paradox that can lead to the non-consideration of the return on public capital in the calculation of the GDP (question mentioned on p.92 of the book) is to picture the following situation (evoked in chapter 16, p.451-452).

Let us imagine a government that owns the equivalent of 100% of GDP in public assets (schools, hospitals…) and has a debt representing 100% of GDP (this public debt being entirely held by residents, to simplify), hence a public wealth equal to zero. Let us assume that the growth is zero, the inflation is inexistent and the effective interest rate is 5% of this public debt, therefore the government disburses 5% of GDP every year in interests to the holders of this public debt, which the government finances with a primary surplus representing 5% of GDP (expenses are every year inferior to the taxes by 5% of GDP)

Let us imagine now that the government decides to privatize completely the public assets in order to reimburse the debt, without any substantial modification occurring in the structure of the economic production: the holders of the debt now own the schools, the hospitals etc; the teachers, the doctors etc. are employed at the same salary as before and perform the same services; the only difference is that there is no debt interests to pay anymore but instead rents to the public assets owners (either through taxes, if the services are still delivered by the government, or through direct payments if they were privatized). Let us assume further that the equilibrium rate of return on capital is always 5%, thus rents represent 5% of the GDP. The difference is that the value of those services, as evaluated in the calculation of GDP, is now increased by 5%, and so is global GDP, in spite of the fact that the set of goods and services produced in this economy has not changed at all. One solution to correct for this bias (which can slightly distort the comparisons among countries) is to apply an average flat rate on the public assets.

Table 2.4. Employment by sector in France and the United States, 1800–2012 (p.91)

This table, replicated on p.91 of the book is based on data from census and enquêtes emploi (Labour force surveys) from census and current population surveys (United States). All the details are indicated in the corresponding excel file.

Data for 1800: France: C. Marchand, O. Thélot, *Le Travail en France 1800-2000*, Nathan, 1997, Tableau A3, p.214 (data of 1806); United States, *Historical Statistics of the United States*, US Census Bureau, 1976, Part 1, p.139 (data on total employment and agricultural employment of 1800; estimates of the decomposition industry/services from incomplete data 1800-1840).

Data for 1900 and 1950: T. Piketty, "Les emplois de services de France et aux Etats-Unis", *Note de la Fondation Saint-Simon*, 1997, Tableau III, p.11 (all the details on

the sources are indicated on p.43-44; numbers for the United States refer to 1900 and 1950, the numbers for France refer to 1906 and 1954).
Data for 2012: from enquête emploi 2011 (France)

Average income in the 18th century and 19th century (p.174-175)

I specify on p.174-175 approximate average income per capita in force in the United Kingdom and in France at different times of the 18th and 19th centuries. All these numbers are extracted from the historical national accounts collected in the database of the *Capital is Back...*, 2013 (see in particular Table UK.2 and Table FR.2).

Figure 2.6. Inflation since the Industrial Revolution

The series used to construct Figure 2.6 replicated on p.108 of the book, are available on Table S2.5, as well as in the corresponding excel file.

Table S2.5. Inflation in rich countries since the Industrial Revolution
(series used for Figure 2.6)

Inflation rates (consumer prices indices) indicated in table S2.5 are also based on the database of *Capital is Back...*, 2013 (see in particular Table US.2, Table DE.2, Table UK.2, Table FR.2).

Annex to chapter 3: The Metamorphoses of Capital

The series used to construct Figures 3.1-3.6 and Table 3.1 replicated on p.116-125 of the book are available on Table S3.1-S3.2, as well as in the corresponding excel file.

Table S3.1. Capital in the United Kingdom, 1700-2010
Table S3.2. Capital in France, 1700-2010
(series used for Figures 3.1- 3.6)

These summary tables are from *Capital is Back...*, 2013 (see in particular Table UK.6f et Table FR.6f). All bibliographic and methodological details are provided in this research (see in particular annexes E and F dedicated to France and the United kingdom).

It must be noted that these estimates over a very long period should be viewed as orders of magnitudes and indications for the general evolutions. Their numerical precision is by nature approximate, in particular for the 18th-19th centuries. In particular, raw materials available do not allow a precise comparison among the different sub periods of the 18th and 19th centuries. To sum up, all the available estimates for these two centuries suggest capital/income ratios between 6 and 8, with no clear trend during these two centuries, in spite of considerable transformations in the nature of capital (land capital being progressively replaced by real estate, industrial and foreign capital holdings). This key conclusion appears robust, but one should not try to go too much beyond this general observation. Thus, though we retained average estimates for the ratio capital/income (around 7), this does not entail that this ratio would have been rigorously steady over the two centuries. The data that we collected do not permit to conclude with certainty about short-run or even medium-run fluctuations.

Similarly, available data do not allow precise comparisons between France and the United Kingdom (except regarding the strongest accumulation of foreign assets in the United Kingdom in the 19th century and during the *Belle Epoque*, and the fastest fall of the agricultural land value). As an example, the land value in the 18th centuries appears a little higher than in France (which could be coherent with the highest demographic density), but it is difficult to be certain. This point deserves to be more deeply analyzed.

Regarding the financial redistributions, and particularly the cancellation of public and private debts, caused by the French revolution, see P. Hoffman, G. Postel-Vinay et J.L. Rosenthal, *Priceless Markets - The Political Economy of Credit in Paris, 1660-1870*, The University of Chicago Press, 2000. See also, J.L. Rosenthal, *Fruits of Revolution. Property Rights, Litigation, and French Agriculture, 1700-1860*, Cambridge University Press, 1992.

Certain limitations of the available data also concern more recent periods. Among the numerous imperfections of the decompositions presented here, let me note that the housing capital only regroups the housing owned by the households (the housing capital owned by companies and public administrations are included in “other domestic capital”). In France, around 85% of housing was owned by households in the early 2010s, compared to about 70% in the early 1970s, which partly explains the increase of the housing share. This point would deserve to be deepened. This limitation of available data has no impact on the overall stock of capital.

Figures on the foreign capital incomes in 1900-1910 (p.121)

I indicate on p.121 that the foreign capital net income reached about 10% of the national income in the United Kingdom in 1900-1913, and about 5% in France for the same period, which equals to the global industrial production of the eastern and northern departments. For the historical series on foreign capital income, see *Capital is Back...*, 2013 (see in particular Table UK.8, Table FR.8). Inflows of foreign capital income reached 9-10% of the United Kingdom national income on the eve of the First World War, for just about 1% of the national income in outflows, thus foreign capital net income of the order in the order of 8-9% of the national income. In France, the foreign capital net income is at the same time around 4-5% of the national income, which is consistent with the fact that foreign net assets are almost twice as high than in the United Kingdom. Also, it must be noted that industry represents in 1900-1910

about 30% of the French GDP (see table 2.4). Northern and eastern departments regroup around 10% of the global population in this period, [9] however on the basis of detailed census results from 1901-1906-1911, we can estimate that they regroup between 15% and 20% of the national industrial production, thus between 4,5% and 6% of the GDP.

Figures on the commercial deficits in 1900-1910 (p.121)

I also write on p.121 that the very high net foreign capital income allowed the United Kingdom and France to finance substantial commercial deficits during the *Belle Epoque.* while improving their wealth position vis-à-vis the rest of the world. For historical series on the foreign net income, see *Capital is Back...*, 2013 (see in particular Table UK.12b, Table FR.12b). If we consider the trade balances average over the periods 1880-1914 or 1890-1914, we can observe a negative trade balance strictly negative in both countries, around -1%/-2% of the national income. On the opposite, the balance of payments, after taking into account the foreign capital net income, is strongly positive. Furthermore it is possible that the average commercial deficit is under estimated for this period, see discussion in *Capital is Back...*, 2013 (see annex France). It is important to note further that the situation of structural commercial deficit did not stop the two countries from experiencing income surpluses during this period. Similarly, that situation of average commercial deficit does not imply that France and the United Kingdom were in an situation of deficit toward the rest of the world, and in particular toward each of their colonies. There are colonies in which France and the United Kingdom exported more than they imported. This only means that in average, their foreign assets allow them to have a structural commercial deficit. [10]

[9] See Annuaire Statistique de la France, Supplément Rétrospectif 1961, pp.14-17: Nord + Pas-de-Calais + Ardennes + Meurthe-et-Moselle + Meuse = around 4,1 millions persons over 39,6 millions in 1911, thus 10,4%; with the 1,8 million inhabitants of Alsace-Moselle (in Germany then), we would have 5,9 millions over 42,4 millions, thus 13,9%.

[10] This question of deficit vis-à-vis this or that colony was often very controversial. I did not try to isolate and study separately commercial balances with the colonies which cause a certain number of difficulties. An important reference (unfortunately biased and incomplete) on these questions is Jacques Marseille's thesis on: « Empire colonial et capitalisme français (années 1880-années 1950) - Histoire d'un divorce », Université Paris I, 1984. The principal message is that the colonial empire soon strained French growth, because it lets age domestic industries survive thanks to their protected exportations to the colonies ("the argument of looting does not stand it is on the contrary the colonies that strained us" he basically says). The problem is that this work does not try to replace the question in the larger context of the global trade balance of France (which is actually in deficit during the Belle Epoque and not positive) and its wealth position (which grows strongly). It does not consider which part of the commercial surplus vis-à-vis certain colonies was being consume on the ground. Neither does it look at in which proportion French imports strained the development of several modern sectors in the colonies. See also the critics wrote by Elise Huilery of J. Marseille's work on the financial balance sheet of French colonialism.

References to the detailed annual series on the ratio capital/income (p.123-125)

I refer on p.123-125 to detailed annual series on the national income and the average private wealth per capita, and on the public assets and debts. These series are available in *Capital is Back...*, 2013 (see in particular Tables UK.1 et UK.6a, Tables FR.1 et FR.6a).

Problem of measure of the financial and non-financial public assets (p.124)

I indicate on p.124 that the frontier between financial and non-financial public assets is not always perfectly clear, illustrating this with this example of the public assets placed in the administration of PTTs (French former postal and communication services) and then in the company "France Telecom". More precisely, the main difficulty concerns the valuation method of such assets. As regards quoted financial assets (for instance after the introduction on the market of part of the France telecom's stocks), the value retained is simply the market value. When the assets are held by an administration, or more generally a non-quoted legal structure (non-quoted companies, autonomous operator of public services, agencies, etc.), such assets are usually accounted for as financial assets (the retained criteria to be treated as a company in the sense of national accounts is to have separate accounting and the fact that the proceeds of sales covers at least 50% of the activity costs), but the valuation method is more complicated: it is based either on the valuations observed for similar quoted companies (that sometimes do not exist), or on the direct estimate value of the of the non-financial asset held by the administration or the structure in question (which can sometimes create problems, see a discussion below on the British public assets). For a methodologically detailed analysis on these questions, see *Capital is Back...*, 2013 (see in particular annex A).

Note on the level reached by British public debt around 1810-1830 (p.129-131)

I write on p.130-131 that the British public debt reached two years of national income by the end of the Napoleonic wars. This well-known fact calls for explanations. The series of the British public debt indicated in table S3.1 and used for figures 3.3 and 3.5 correspond to the nominal value (or face value) of the public debt. If we used the market value (which would be more conform with the current norms of national accounting), then we would observe that the public debt value is lower in the 1810s

(around 100%-150% of the national income), and reaches its highest level a little after, in the 1820-1830s (around 180%-200%of the national income). See *Capital is Back...*, 2013, Tables UK.5e and UK.6e for detailed series.

The reason can be explained as follow. By the end of the Napoleonic wars, the British state had to pay very high interest rates on the public debt (in the order of 6%-7%). As a result, it sells its debt equity a lot less under par value: for instance a bond from which can be expected a 100£ reimbursement and a nominal interest rate of 4% is sold strictly under 100£ (hence a real interest rate higher than 4%, typically 6%-7%). This explains why the public debt market value is below its nominal value in the 1810s. While the interest progressively decrease to 3%-4% in the 1820s-1830s, the debt's market value rise toward its nominal value. Finally, the main point is that the British public debt equity holders will become owners in the 1820s-1830s of a debt worth 200% of the national income, whereas they lent far less during the previous decades (in effect, they made a great deal by betting on the state whose credit had temporarily diminished). These evolutions, and these difficulties due to the public debt value, are analyzed in more detail in *Capital is Back...*, 2013 (see in particular annex F).

Note on the budget surpluses in the United Kingdom, 1815-1914 (p.131)

I specify on p.133 that the average primary budget surplus is between 2% and 3% of the United Kingdom national income from 1815 to 1914, and financed public debts interest of a similar amount (or slightly higher, hence a small secondary deficit), which globally represents more important masses than the global education budget. For these detailed annual series on the budget balance and the debts interest, see *Capital is Back...*, 2013, Table UK.12c. On the education budgets for this period, see chapter 13, p.475, note 2.

Note on the public debt and the public deficits in France, 1815-1914 (p.132)

I write on p.132 of the book that the increase of public debt in France in the 19th century is explained in a large part by a series of exceptional disbursements. The compensations paid to the occupation armies in 1815-1816 were around 2 billion gold francs, which amounts to 3 billions after adding the "emigrants billion" in 1825, at a time when national income was between 8 and 9 billion (i.e. around 30%-35% of the national income). Compensations paid in 1871-73 were 7,5 billions gold francs (2,5 billion in occupation fees, and 5 billion of pure transfer), in a period in which the

national income was between 21 and 22 billions (around 30%-35% of national income). These two transfers explain by themselves the main part of the public debt increase. See *Capital is Back...*, 2013, DataFR2.[11]. Finally, in the course of the 1880-1914, the public debt interests represent on average around 2%-3% of national income in France. For annual series, see *Capital is Back...*, 2013, Table FR.11b

Note on the estimates of the public assets in the post-war United Kingdom (p.138)

I write on p.138 that the public capital turns positive in the United Kingdom in the 1960s-1970s (after the public debt was reduced by inflation), but does not reached the level it reached in France. It is important nonetheless to indicate that it is difficult to make a precise and satisfactory comparison. In particular, it is possible that the British public assets value indicated in table S3.1 (and used for graphs 3.3 and 3.5) underestimates significantly the British public assets value in the 1960s-1970s, especially if we consider the levels reached after the 1980-1990s privatizations. The available estimates concerning the residual value of the British public companies (difference between the accounting value of their assets and the registered value as public assets) suggest that the net public capital achieved 70%-100% of the national income in the 1970s in the United Kingdom (and not barely 20%-30% when we ignore this residual value). See *Capital is Back...*, 2013, Table UK.6a and annex F). In this case, the public capital reached 20%-25% of the British capital in the 1970s (and not less than 10%). It is therefore possible that the difference in level with France be less important that what indicates the comparison of Figures 3.3-3.6, and that the gap between the two countries mainly concerns the timing (it took the United Kingdom several decades to reduce its public debt inherited from wars, thus the net public capital becomes positive only at the end of the 1960s and during the 1970s, whereas it is highly positive in France as of the 1950s and stay as it is during the entire “Thirty glorious years” (so was the situation in Germany; see chapter 4). [12]

[11] On those transfers, see also L. Fontvieille, "Evolution et croissance de l'Etat français, 1815-1969", Economies et sociétés, 1976, p.1860-1868.

[12] It is difficult to conclude precisely at that point, because the data sources are not perfectly comparable. In particular, only the British accounts include complete historical estimates separated for the public companies residual value (and not only for all the companies)

Appendix to Chapter 4: From Old Europe to the New World

The series used to construct figures 4.1-4.3, replicated in the book on p.141-144, are available in table S4.1, as well as in the corresponding excel file.

Table S4.1. Capital in Germany, 1870-2010
(*series used for figures 4.1, 4.2 and 4.3)*

This summary table has been taken from *Capital is Back...*, 2013 (in particular Table DE.6f). All the bibliographical and methodological details are given in this research (in particular appendix D dedicated to Germany, where the method to treat the numerous territorial changes, in order not to bias the evolutions, is explained in details.

The series used to construct figures 4.4-4.5, replicated in the book on p.145-147, are available in table S4.1, as well as in the corresponding excel file

Table S4.5. National, public and private capital in Europe and the United States, 1870-2010 (*series used for figures 4.4 and 4.5)*

This summary table has been taken from *Capital is Back...*, 2013 (in particular Table A2). The European average is a computation of the average of United Kingdom, France, Germany, and Italy from 1970. The general shape of the curve is very close in the different countries, thus including another country, or doing simple arithmetical average rather than weighting with the national income of each country, has a limited impact on the European average (refer to Table A2 for alternative calculations). Including Spain, where the capital/income ratio has surged even more than in other big European countries since the 1970s-1980s (but complete official series only started in 1987), would reinforce the increase of the European ratio. Series regarding Italy (from 1970) and Spain (from 1987) are analyzed in chapter 5.

Furthermore, even though we did not try to establish series for the whole 20th century for European countries other than the United Kingdom, France and Germany (due to the lack of satisfactory sources), available estimates for Italy around 1900-1910, for example, also suggest a capital/income ratio with a magnitude of six-seven at the Belle Epoque, as much as in the three biggest countries. The partial estimates that are available for Belgium, Holland and Austria lead to the same results. Refer to *Capital is Back...*, 2013, technical appendix, for bibliographical references. However, we cannot exclude the fact that some European countries followed substantially different paths from the ones observed in the three biggest countries, with "U-shaped curves" much less pronounced in the 20st century for instance, due to a smaller effect of the shocks linked to the wars (for example, see the Swiss case studied by Dell-Piketty-Saez 2007); or with capital/income ratio structurally smaller in the 1900-1910, due to the small values for industrial capital, net foreign capital (sometimes negative) or the value of lands (abundance of the lands compared to the population, like in North America), as the available estimates seem to suggest for Sweden (see work in progress of Roine and Waldenstrom)

Note on the fall capital/income ratio in Europe, 1913-1950 (p.146-147)

I analyze p.146 the different factors explaining the fall of the capital/income ratio in the United Kingdom, France and Germany between 1913 and 1950, and especially the relative importance of the destructions due to the wars, the small savings, the fall of the relative prices of financial and non-financial assets. More detailed breakdown are presented in *Capital is Back...*, 2013 (in particular Table 10 and Tables A108-A137).[13]

Figure 4.6. Capital in the United States, 1770-2010 (**p.151**)
Figure 4.7. Public wealth in the United States, 1770-2010 (**p.153**)
Figure 4.8. Private and public capital in the United States, 1770-2010 (**p.154**)
Figure 4.10. Capital and slavery in the Unites States (**p.160)**
Figure 4.11. Capital around 1770-1810: Old and New World (**p.161**)

The series used to construct figures 4.6-4.8, replicated in the book on p.151-154, are available in tables S4.2 et S4.4, as well as in the corresponding excel file.

[13] Regarding the French case, also refer to *On the Long-Run Evolution of Inheritance...*, 2010, Appendix A, pp.42-43 (and QJE 2011 p.1094), where I indicate additional decompositions between destructions and foreign portfolios losses. Also see *Les hauts revenus en France...*, 2001, p.137. The most complete estimates are presented in *Capital is Back...*, 2013.

Table S4.2. Capital in the United States, 1770-2010
Table S4.4. Capital and slavery: Old and New World, 1770-1810
(*series used for figures 4.6, 4.7, 4.8, 4.10 and 4.11)*

These summary tables have been taken from Capital is Back..., 2013 (in particular Table US.6f). All the bibliographical and methodological details are given in this research (in particular appendix B dedicated to the United States, where all the sources that are used are described, especially to estimate the market value of slaves, from the estimates of prices and number of slaves; on this subject also see Table US.3b).

Figure 4.9. Capital in Canada, 1860-2010 (**p.157**)
Figure S4.1. Public wealth in Canada, 1860-2010
Figure S4.2. Private and public capital in Canada, 1860-2010

The series used to construct figure 4.9, replicated in the book on p.157, and supplementary figures S4.1-S4.2, are available in table S4.3, as well as in the corresponding excel file

Table S4.3. Capital in Canada, 1860-2010
(*series used for figures 4.9 and S4.1-S4.2)*

This summary table has been taken from Capital is Back..., 2013 (in particular Table CA.6e). All the bibliographical and methodological details are given in this research (in particular appendix H dedicated to Canada).

Appendix to Chapter 5: The Capital/Income Ratio over the Long Run

Figure 5.1. Private and public capital: Europe and the US, 1870-2010 (**p.164**)
Figure 5.2. National capital in Europe and in America, 1870-2010 (**p.165**)
Figure S5.0. Private capital in Europe and in America, 1870-2010

The series used to construct figures 5.1-5.2, replicated in the book on p.164-165, as well as supplementary table S5.0, are available in table S4.5, as well as in the corresponding excel file (see Appendix to Chapter 4).

Note on the law β=s/g (**p.166-168**)

The law β=s/g, of which the intuition is explained in the book on p.166-168, stems directly from the basic mathematical equation describing wealth accumulation. In a model without price effect, and where wealth entirely comes from accumulation (no natural resources), wealth in year t+1 W_{t+1} simply equals the sum of wealth in year t W_t and savings S_t :

$$W_{t+1} = W_t + S_t$$

If we divide each term by national income in year t+1 Y_{t+1}, and if we call $\beta_t = W_t/Y_t$ the wealth/income ratio, $1+g_t = Y_{t+1}/Y_t$ the growth rate of the national income, and $s_t = S_t/Y_t$ the savings rate, we obtain the following equation:

$$\beta_{t+1} = \beta_t \times (1+s_t/\beta_t)/(1+g_t)$$

Intuitively, wealth grows at rate s_t/β_t, thus the wealth/income ratio increases if $s_t/\beta_t > g_t$, and decreases if $s_t/\beta_t < g_t$. This implies that if the savings rate and the growth rate both stabilize at some given level $s_t=s$ and $g_t=g$, then the wealth/income ratio β_t must necessarily converge to β=s/g.

A more detailed analysis of these basic equations, completed with relative price effects, can be found in On the Long-Run Evolution of Inheritance…, 2010, p.32-40, and in Capital is Back…, 2013, p.13-18.

Figure 5.3. Private capital in rich countries, 1970-2010 (**p.171**)
Figure 5.4. Private capital measured in years of disposable income (**p.181**)
Figure 5.5. Private and public capital in rich countries, 1970-2010 (**p.184**)

Figure 5.6. Market value and book value of corporations (**p.189**)
Figure 5.7. National capital in rich countries, 1970-2010 (**p.192**)
Figure S5.2. Private capital in rich countries: from Japanese to the Spanish bubble

The series used to construct figures 5.3-5.7, replicated in the book on p.181-192, as well as the supplementary figure S5.2, are available in tables S5.1-S5.3, S.5 and S5.11-S5.12, as well as in the corresponding excel file.

Table S5.1. Private capital in rich countries, 1970-2010
Table S5.2. Public capital in rich countries, 1970-2010
Table S5.3. National capital in rich countries, 1970-2010
Table S5.5. Foreign capital in rich countries, 1970-2010
Table S5.11. Ratio between market value and book value of corporations in rich countries 1970-2010 (Tobin's Q)
Table S5.12. Private capital/disposable income ratio in rich countries, 1970-2010
(series used for figures 5.3.5.7 and S5.2)

All of these tables have been taken from *Capital is Back...*, 2013 (in particular Tables A1-A12). The methodological details are available in the technical appendix of this research. The same applies for the following figures on the structure of savings, reproduced in the book p.174-186 and taken from Tables 2-3 et A82-A98 of *Capital is Back...*. One should notice that most of the series covering the period 1970-2010 have been taken from the official national accounts of the different countries.

Table 5.1. Gross rates and saving rates in rich countries, 1970-2010 (**p.174**)
Table 5.2. Private saving in rich countries, 1970-2010 (**p.177**)
Table 5.3. Gross and net saving in rich countries, 1970-2010 (**p.178**)
Table 5.4. Private and public saving in rich countries, 1970-2010 (**p.186**)

Note on the predicted and observed private capital in 2010 (**p.175-176**)

I write on p.175-176 that the observed private capital in 2010 in rich countries is relatively well predicted by the accumulation of savings and investment flows between 1970 and 2010. On this subject also see the following figure and table:

Figure S5.1. Accumulation of private capital in rich countries, 1970-2010
Table S5.4. Predicted and observed private capital in rich countries, 1970-2010

For a detailed analysis of this question, see *Capital is Back...*, 2013 (in particular Tables 4-5 and Figures 7a-7b). The next two points are worth to be emphasized. On the one hand, it is more satisfactory to study this question with respect to national capital accumulation instead of only taking into account private capital. On the other hand, even if the overall evolution of the capital stock in the long run is well explained by the accumulation of savings flows, one should not exaggerate this fact. In particular, one should not forget the importance of price effects, whether it be the long-term catching-up of asset relative prices that occurred in most of the countries between 1970 and 2010, or asset price variations, sometimes erratic, between countries (which tend to compensate when we look at averages in sufficiently large economic areas, at the European level for instance, see Figures 7a-7b).

Note on durable goods, valuables and gold (**p.179-180**)

I write in the book on p.179-180 that durable goods (not included in official wealth accounts) generally account for between 30% and 50% of national income, and that this level seems to be relatively stable: during the period 1970-2010 as well as on the long the run, from the 18th to the 21st century. For detailed series, see *Capital is Back...*, 2013, Table US.6f, Table DE.6f, Table FR.6f, and Table UK.6f, etc.

I also write on p.179-180 that currently, valuables and precious metals (including gold and coins) are barely worth 5-10% of the national income, and have been decreasing on the long run. For the current period, see for instance the detailed version of the French official wealth accounts.[14] We notice that the valuables and precious metals owned by the households (code AN.13) amount to 115,4 billion euros on 12-31-2010, and the monetary gold and the Special Drawing Right owned by financial companies (code AF.1) amount to 93.9 billion euros, or around 5% of GDP in each case.

If we try to calculate the value of the stock of gold in the world (see in particular the data of the World Gold Council available here), we obtain the following results. The global stock is around 120 000 tons of gold, of which more than two thirds are jewelry, a bit more than one tenth in industrial and medical applications and around one fifth in the form of reserves of the central banks.[15] The price of one kilo of gold being 35,000€ in 2012-2013, it amounts to more than 4,000 billion euros, almost 6%

[14] The same kind of tables with raw data are available in this directory.

[15] We estimate that 145 thousand tons of gold has been extracted since the Antiquity, but that 25 thousand tons were lost (gold is not totally hard-wearing). Currently, 2.5 thousand tons are extracted every year (less than 0.1% of the global GDP, fewer than gas and mining extractions; see chapter 12), which is not sustainable forever (the reserves estimates are 50 thousand tons)

of the global GDP (around 1.5% of the global wealth). We notice a strong increase in the stock market prices since 2007-2008 (due to the safe haven status of gold). The variations since 1945 are relatively erratic, with a downward trend during the Bretton Woods period, a rise during the 1970s, then a decrease from the 1980s to the middle of the 2000s. We can estimate the global stock of gold was worth 2% of the global GDP around the 1970s.[16]

If we go back in time, we notice a decrease on the long run: the stock of gold was worth around 20% of national income in the 19th century. In particular, according to the estimates gathered by Raymond Goldsmith (*Premodern financial systems*, Cambridge University Press, 1987, table 4.3 p.58), we can estimate that monetary metals amounted to around 10%-20% of the GDP in France, in Germany, in the United States, and in the United Kingdom between 1800 and 1910 (thus around 2%-3% of the total wealth). Goldsmith also gives an estimate of the stock of gold in the Roman Empire amounted to 20% of GDP.[17] Gregory King estimates the value of the stock of gold and precious metals for the United Kingdom around 1700 to be between 20% and 30% of the national income of the period. For a detailed description of the estimates of King, see *Capital is Back...*, 2013, technical appendix p.109-114.

[16] It is interesting to notice that the price of gold in $ barely changed between 1792 and 1932 (around $19-20 per ounce, or approximately 30g); then it was $35 per ounce between 1935 and 1970 (Bretton Woods period; we already noticed a peak at $40 in 1968-1969). Then it was up to around $200 in 1975, then it rose above $600 in 1980, before stabilizing around $300-400 from 1984 to 2004, then rose up to 700$ in 2007 and $1000 in 2010 then more than $1,500 in 2012. Over the period 1970-2012, the price was multiplied by 50, whereas global GDP was multiplied by 25 (from $3tr to $70tr). Given this increase in the quantity, the value of the total stock of gold went from around 2% of the global GDP in 1970 to more than 5% in 2012 (but was only 1% at the beginning of the 2000s). The right way to decompose the historical evolution of the price of gold is the following one: o=qO/pY (where q = price of gold, p= general price index, O= quantity of gold, Y = real global GDP. It has to be noticed that gold would not have been an international currency for a long time because its stock was not growing as fast as the global GDP (see discussion in chapter 16, p.548 in the book). With the Gold Standard, that is to say with q=1, O has to increase as much as Y in order to maintain o constant. The problem is that there is no link between the gold discoveries and the growth rate. It is a classic explanation of the Great Deflation of 1870-1900: the stock of money increases less fast than the production, i.e. O/Y decreases, thus prices decrease (so that o=O/pY almost remains constant). Currently, O increases by 2% per year, this is a fast rhythm, but smaller than the world growth rate. Moreover, this path can be erratic, as the 19st century shows.

[17] According to Goldsmith, the value of the stock reached 30% of the GDP in Florence in the 15th century, or even 90% in India around 1860, and more than 100% in Athens including all the public treasury (or almost 15% of the wealth at this period). Refer to R. Goldsmith, op.cit, p.23. These estimates would be worth being closely re-examined.

Note on foundations and other holders of capital (**p.182-183**)

I write in the book on p.182-183 that the share of foundations and other non-profit associations in the total wealth is always less than 10% and generally less than 5% in rich countries in the 1970s-2010s, with however some interesting variations (barely 1% in France, 3-4% in Japan, 6-7% in the United States). For the detailed series, refer to *Capital is Back...*, 2013 (in particular Table A65). The detailed decomposition on an annual basis is only available for the United States, Japan and France. For other countries, assets and liabilities owned by "non-profit institutions serving households"(NPISH) (according to the official categories in national accounts) are directly included in the household sector. More generally, the fragility of these data has to be highlighted and additional research would be required. I also give on p.182-183 an estimate for the goods owned by the Church during the Ancient Régime: around 50%-60% of the national income in 1789 (around 2.4 billion pounds of national goods according to Sargent-Velde JPE 1995 p.496, when the national income is around 4.5-5 billion pounds). Furthermore, Sargent-Velde JPE 1995 p.485 indicates that the goods owned by the Church amounted to 10%-15% of the total wealth, but it seems only to refer to rural properties. If we take into account the whole national wealth (around 700% of the national income according to our estimates), we are closer to 7-8%. It should also be noted that national goods do not include only goods owned by the Church and that Sargent-Velde do not give any additional decomposition.

Note (book value)/(market value) ratio (Tobin's Q) (**p.187-190**)

In addition to the structural factors presented in the book (p.187-190), purely statistical reasons can explain why the Tobin's Q is structurally superior or inferior to one in a given country in a given period (for instance if the investment and/or the depreciation are systematically under-evaluated or over-evaluated). That's why I think it is better not to include the residual value of the firms (difference between the book value and the market value) in the definition of the national wealth. Market values can be more easily compared between countries and, moreover, are available for many more countries and over longer periods (book and residual values are only available for recent decades and for a small number of countries).

In any case, including this residual value would lead to a reduction in the national wealth of some countries (for instance the United Kingdom) and an increase for some others (for instance Germany), but this would not significantly modify the overall long

term evolution (specially, this would have almost no effect on the European average). For a more detailed discussion on these questions, refer to *Capital is Back...*, 2013.

Note on the German surplus (**p.191**)

I compare on p.191 the magnitude of the German trade surplus reached in the recent period and the French financial and non-financial capitalization. See Tableau S5.13 for trade balances of different rich countries during the period 1070-2010 (see *Capital is Back...*, 2013, for detailed annual series per country). Besides, the valuation of the Parisian real estate is 15% of the country real estate (which is around 6,000 billions euros according to the national accounts), thus approximately 800-900 billions euros.[18]

Note on the structure of financial assets and liabilities (**p.192-193**)

I mentioned in the book on p.192-193 the surge of total financial assets and liabilities in rich countries during the period 1970-2010, and in particular, the rapid growth of cross-investments between countries. The following figures and tables, which have all been taken from *Capital is Back...*, 2013, and have not been integrated in the book (in order to limit its size), enable the reader to more precisely realize the magnitude of the phenomenon.

[18] The housing survey gives a figure closer to 10%, but data on inheritance give something closer to 15%. It does not dramatically modify the order of magnitude for comparison.

Table S5.10. Foreign liabilities/total financial liabilities in rich countries, 1970-2010
(series used for figures S5.3-S5.11)
Table S5.13. Foreign assets accumulation in rich countries, 1970-2010: balance of payment, trade balance and capital income

To analyze the importance of capital gains or losses made by the different countries in their international portfolio (with specially huge capital gains for the United States, and huge capital losses for Germany), also see *Capital is Back...*, 2013, Table 6, and the corresponding text.

If we look at very long periods, we notice that the magnitude of the financialization trend of these last decades is much bigger than what happened before in the financial sector. In particular, the historical financial data gathered by Raymond Goldsmith (*Comparative National Balance Sheets*, The University of Chicago Press, 1985; for example this table) show that the value of financial assets in Western countries rose from less than 100% of the national income around 1700 to more than 300% around 1800, then 500-600% around 1900-1930, before stabilizing at this level during 1930-1980. Since the 1980s, it has been risen up to 1,000%-2,000%, or even more if we take into account derivatives.

Figure 5.8. The world capital/income ratio, 1870-2100 **(p196)**

The series used to construct figure 5.8, replicated in the book on p.196, are available in Table S12.4 (see appendix to chapter 12). All the details about the assumptions on which the series are build, especially for the period 2010-2100, are specified in the book, as well as in the corresponding excel file. As for Japan before the 1970s, the complete inheritance data that started in 1905 were collected by Morigushi and Saez (in particular table B1). These data show a huge fall in the Japanese inherited wealth between the 1920s-1930s and the 1950s-1960s and a (incomplete) rise since. It is not possible to properly disentangle the aggregate effect from the concentration effect, but this seems very close to the European evolutions.

Note on the value of land (p.196-198)

I write on p.196-198 that it is difficult to isolate the value of “pure land” (land prior to any human improvements). In particular, I write that pure land in traditional rural societies generally accounts for a small portion of the total value of farmland, once the cumulated value of the investments made has been withdrawn (although it is hard

to give an exact figure). The same problem arises with urban land. For example, if we look at the official wealth accounts of France, we notice that the value of land rose from one year of GDP in 1980, 1990, 2000 to almost three years in 2010 (mainly due to urban land)[19]. But this is mainly due to the estimate method: the French national accounts attribute all the increase in the real estate prices to the land. But the initial level of the real estate prices in the 1970s (when the French national accounts started) was historically low, so that a big part of the capital gains (between a half and three quarters according to our estimates) should be attributed to buildings. In the end, according to the decompositions of wealth accumulation on long period presented in *Capital is Back...*, 2013, Tables 8-9, it seems that the capital stock in 2010 can be explained mainly by the accumulation of saving flows during the period 1870-2010. Consequently, the value of pure land (rural or urban) is smaller than the one indicated by these estimates (probably less than one year of national income). This difficult question of land value raises lots of interrogations and would require many more clarifications.

[19] According to wealth accounts on 12-31-2010, the total value of lands (code AN.211, national economy) reached 5,817.8 billions euros (GDP in 2010 : 1,932.8 billion euros). On 12-31-2000, it was 1,442.8 billion euros (GDP in 2000: 1,439.6 billion euros). On the 12-31-1990, it was 1,073.3 billion euros (GDP in 1990 : 1,032.8 billion euros). On 12-31-1980, it was 432,7 milliards (GDP in 1980 : 444,7 billion euros).

Appendix to Chapter 6. The Capital-Labor Split in the 21st Century

The series used to construct figures 6.1-6.4, replicated in the book on p.200-202, as well as the supplementary figures S6.1-S6.2, are available in tables S6.1-S6.2, as well as in the corresponding excel file.

Table S6.1. The capital-labor split in the United Kingdom, 1770-2010
Table S6.2. Capital-labor split in France, 1820-2010
(series used for figures 6.1-6.4 and S6.1-S6.2)

These two tables have been taken from *Capital is Back...*, 2013 (in particular Tables A49-A50). The interest payments on public debt (not taken into account in national income) are excluded from the capital-labor split presented here. Series with interest payments on public debt are available in the corresponding excel file. For France, series were built from historical data gathered in *On the Long-Run Evolution of Inheritance...*, 2010, Appendix A. For the United Kingdom, series were built from the works of Allen 2007 (in particular figure 2) and 2009 for the period 1770-1910 and from modern national accounts (Feinstein's series then official series) for the period 1910-2010. One of the difficulties is due to the fact that Allen's series give a capital share 5-10% higher than the one for the recent series, which can be explained in particular by a different way to take into account non-wage labor. We homogenized his series with the previous ones to avoid discontinuities. As a result, capital shares for 1770-1910 may be too low. As explained in the book, measuring the capital share over long periods raises a lot of difficulties and questions. All the details on the imperfect series presented here are given in this excel file.

Note on the evolution of interests on public debt (**p.207**)

I write on p.207 that the interest rate on the public debt is typically around 4%-5% per year during the 18th and 19th centuries. One of the most documented cases is the United Kingdom, where we have quite complete annual series started at the end of

the 17th century and the beginning of the 18th century. We notice that returns often reached 5%-6% in the 18th century, or even 6%-7% at the end of this century and at the beginning of the 19th century (and during the Napoleonic wars; see appendix to chapter 3), then progressively decreased again during the 19th, to eventually be barely 3%-4% at the end of the century (or even less than 3%, while in a deflationist context, so that the real returns were in fact significantly higher). See series gathered in *Capital is Back...*, 2013 (in particular Table DataUK4). The series that are available for other countries, specially France and the United States, are less systematic but show the same kind of evolutions and fluctuations around an historical mean about 4%-5% in the 18th and 19th century.[20]

Note on the importance of non-interest-bearing checking accounts (**p.209**)

I write on p.209 that although the major share of the savings of a large part of the population is held in non-interest-bearing checking accounts, the latter only account for 10-20% of national income (that is, 3-4% of the total wealth). Let's take the example of official wealth accounts for France.[21] On 12-31-2010, all the "currency and deposits" (code AF.2) held by households amount to 1,123.3 billion euros (around 55% of GDP), including 49.3 billion in "banknotes et coins" (AF.21, 2% of GDP), 288.9 billion in transferable deposits (AF.22, around 15% of GDP; it is the part that corresponds to checking accounts), et 785.1 in "other deposits" (AF.28 et AF.29, around 40% of GDP, including one third in contractual savings products – French *livret A* for example- and two thirds in "sight deposits" – typically savings account bearing interests.

Note on the relationship between the production and the capital share (**p.215-220**)

I explain in the book on p.215-220 the relationship between the production function and the capital share through the concept of elasticity of substitution. The corresponding basic equations are the following ones.[22]

The simplest case is the Cobb Douglas production function:

[20] On the evolution of returns to the American public debt payable after ten years from 1790 to 1914, also see *EEAG Report on the European Economy*, Ces-Ifo, 2013 (dedicated to a comparison of the implementation of federal debt in the United States and in Europe). The American returns decrease from 6%-7% in 1790-1810 to 5%-6% during the period 1810-1870, then 3%-4% during the period 1870-1910. See p.98, figure 4.1.

[21] See this table. The same kind of table with raw data for other countries are available here.

[22] For further information, the interested reader can refer to my lecture notes.

$$Y = F(K,L) = K^{\alpha} L^{1-\alpha}$$

In this case, the marginal productivity of capital is given by:

$$F_K = \alpha K^{\alpha-1} L^{1-\alpha} = \alpha Y/K = \alpha/\beta$$

(where $\beta = K/Y$ is the capital/income ratio)

Consequently, if the return to capital is determined by the marginal productivity of capital, that is $r = F_K$, then the capital share $r \times \beta$ is always exactly equal to α (thus a purely technological parameter), independently from β.

Let's consider now the case of a production function characterized by an elasticity of substitution $\sigma \geq 0$ (such a function is called CES, constant elasticity of substitution):

$$Y = F(K,L) = [\, a K^{(\sigma-1)/\sigma} + (1-a) L^{(\sigma-1)/\sigma} \,]^{\sigma/(\sigma-1)}$$

In this case, the marginal productivity of capital is given by:

$$F_K = a (Y/K)^{1/\sigma} = a \beta^{-1/\sigma}$$

Consequently if $r = F_K$, then the capital share $\alpha = r \times \beta$ is given by:

$$\alpha = r \times \beta = a \beta^{(\sigma-1)/\sigma}$$

To say it differently, the capital share α is an increasing function of β if and only if the elasticity σ is higher than 1. If σ gets closer to 1, then the CES function tends to a Cobb-Douglas function and the capital share get closer to the absolute stability (it does not depend on β any more). If σ tends to infinity, then the CES production function tends to a production function $F(K,L) = r K + v L$, and the returns to capital does not depend on the quantity anymore (completely robotized economy).

These formulas can be used to notice that when the capital/income ratio β varies a lot – like during the period 1970-2010 – the elasticity of substitution just need to be slightly higher than 1, so that the capital share α varies a lot too. For instance, with $a=0.21$ and $\sigma=1.5$, and if the capital/income ratio rises from 2.5 to 5 then 8, then the capital share $\alpha = r \times \beta = a \beta^{(\sigma-1)/\sigma}$ rises from 28% to 36% then 42%. If, meanwhile, the

elasticity of substitution becomes σ=1.8, then the capital share becomes 53% for β=8.

We have just seen that there is no need to assume an infinite elasticity of substitution to conclude that the capital share can easily take extreme values. Given the variations of the capital share observed during the 20th century, and the observed increase in rich countries during the period 1970-2010, we can conclude that this variation can be adequately explained with an elasticity of substitution slightly higher than 1 (1.3-1.6). However, it is obvious that it is extremely simplistic to summarize all the possibilities of substitution between capital and labor (as regards technology as well as consumption patterns) with a sole parameter. Moreover, in any case, the elasticity of substitution between capital and labor in the future may be different (and possibly higher) than the past one. For further and more thorough discussions on these questions, see Capital is Back..., 2013, p.34-37.

Figure 6.5. The capital share in rich countries, 1970-2010 (**p.222**)

The series used to construct figure 6.5, replicated in the book on p.222, are available in table S6.3, as well as in the corresponding excel file.

Table S6.3. Capital share in rich countries, 1970-2010
(series used for figure 6.5)

This table has been taken from *Capital is Back...*, 2013 (in particular Tables A48-A48b). As I explain in the book, the series on the capital-labor split that are currently available are far from being perfectly comparable over time and between countries. The rise of the capital share in rich countries over the past decades seems however relatively robust, as it was reported by several authors and official reports using very different data and accounting standards. To start looking into this massive literature, see for example references indicated by Atkinson 2009, ILO 2012 or Guerriero 2012. The ILO (International Labor Organization) report also shows the difficulties currently encountered to extend the measures and the debate on the capital-labor split to emergent countries (given little available data).

Figure 6.6. The profit share of the value added of corporations in France, 1900-2010 (**p.226**)
Figure 6.7. The share of housing rent in national in France, 1900-2010 (**p.226**)
Figure 6.8. The capital share in national income in France, 1900-2010 (**p.227**)

Figure S6.3. Capital share of the disposable income in France, 1896-2010

The series used to construct figures 6.6-6.8, replicated in the book on p.226-227, and figure 6.3, are available in table S6.4, as well as in the corresponding excel file.

Table S6.4. Capital share in France, 1896-2010
(series used for figures 6.6-6.8 et S6.3)

This table has been taken from *On the Long-Run Evolution of Inheritance...*, 2013 (in particular Appendix A).

Note on the very long term series on land rent (**p.225**)

I write in the book on p.225 that it is also possible to study the capital-labor split with very long-term series on land rent and wages in the agricultural sector, such as the ones used in particular by Jeffrey Williamson and his colleagues. To find references on his works (mainly dedicated to the United Kingdom and the United States) and an extension to the France case, see Rouzet 2005. There is one limitation to this approach: the link with national accounts is not always clearly established (this is more a partial equilibrium approach)

Note on the companies' accounts and the statistics analyzed by Marx (**p.229**)

I write in the book on p.229 that Marx gave some examples of companies' accounts, which can let us imagine the capital intensity he had in mind. The more elaborated statistics that Marx analyzed can be found, for most of them, in Capital (1867), Volume 1, Chapter 25, named " The General Law of Capital Accumulation", or more exactly in the huge appendix 10 to Chapter 25.[23] In particular, Marx used in this appendix the statistics from the British corporate income tax 1846-1866. He intended to show that capital-intensive development is characterized by an explosion of the profits and potentially an increasing concentration of the capital. He wrote for instance that the industrial profits (as well as rent land) rose by around 40%-50% between 1853 and 1864 whereas the population only rose by 12%.[24] However, he

[23] See Capital, Volume 1, p.885-954. I use hereby the pagination of the French edition Folio Essais 2008, which uses the text of the edition of Maximilien Rubel and published in La Pléiade in 1963. Bear in mind that Volume 1 of Capital went out in German in 1867 (and in French in 1875), and that Marx died in 1883 without completing Volumes 2 et 3. The appendix 10 to chapter 25 of Volume 1 is part of the numerous technical developments that were included in the full text of the 1867 edition, but that Engels then Rubel decided afterwards to refer to it in an appendix at the end of the book, on the basis of the fragments and instructions that Marx left for the complete edition.
[24] See Capital, Volume 1, p.886.

did not try to estimate the level and the evolution of the wage bill, of the production and of the national income, and as a consequence he did not tackle the question of the capital-labor split or the capital/income ratio. In particular, he did not try to link the fiscal statistics with the estimates of national capital and national income, which had multiplied in the United Kingdom since the beginning of the 18th century (Petty, King) and in the 19th century (Colquhoun), and which Giffen was about to systematize in the 1860s-1880s (his work is not quoted by Marx). He also gave some information about a possible increasing concentration of profits and wealth but rather anecdotally.[25]

In fact, the data that Marx used and that give the most information and can let us have a better idea of what kind of capital-intensity he had in mind are in the Chapter 9 of the Volume 1 of Capital called "The Rates of Surplus-Value". In particular, Marx gives a detailed example on the accounts of a mill, "given by the owner himself", specifying the whole set of technological and financial aspects, the fixed and variable assets, the number of spindles and workers, as well as the profits and wage bill, the quantity of cotton and coal used for the production, and of course the rent.[26] Even if Marx did not exactly put it this way, it results from the accounts that the capital-intensity of this mill is extremely high, with a K/Y ratio around 10, and a capital share about 50%, or even higher (which corresponds to a profits/wages ratio, or "rate of exploitation" according to Marx's terminology, about 100%, or even more). On the basis of other examples of mills mentioned in this chapter or in other parts of the book, we can consider that it is the kind of order of magnitude Marx had in mind for the industrial capitalism of his time.

[25] For example, he gives on p.887 a table of fiscal statistics that allows us to know the evolution of the profits made by bracket, and he specifies that the higher brackets increased more than the others between 1864 and 1865. This is something interesting, that also shows how comprehensive were the statistics from the Schedular system of taxation enforced at that time in the United Kingdom (see for instance the interesting statistics he gives on p.943 that show the huge size of the rent from land compared to other incomes in Ireland in 1863. But it is obvious that the evolution of the industrial profits over two years does not allow drawing general conclusions on the long-term evolution of the wealth distribution. Marx also note write on p.887 that according to the inheritance statistics published by the British administration since the early 19c, all inherited financial assets were below one million pounds between 1815 and 1825, and four inheritance exceeded this amount between 1856 and 1859. Again, it is an interesting fact, but not enough, and that Marx did not try to explore. However,, it should be mentioned that Marx deserves the credit for at least mentioning these data, whereas some economists at that time, were only producing some pure theoretical speculations.

[26] See on pp.320-321 of Capital, Volume 1. The accounts of this mill are supposed to refer to 1871. (lhe Frenc edition of 1875 is based on the second German edition, published in 1872, in which Marx added a lot more elements)

Note on the Cambridge capital controversy and the growth instability (**p.231**)

I write on p.231 that the equation β=s/g first proposed by Harrod-Domar was initially thought as an expression of the instability of the growth process (assuming β is set by technology, at least in the short run). For theoretical model trying to explain the fact that the equilibrium between savings and investment can generate a persistent instability in the short run, see this article. On the violence of the confrontation between the economists from the two Cambridge until the 1960s-1970s, read for instance this criticism of Pasinetti's book by Stiglitz.

Note on the vagaries of technology (**p.234**)

I write on p.234 that the different hypothesis on technology (especially the elasticity of substitution) can generate very different conclusions on the long-term evolution of the capital/income ratio, the capital share and thus the structure of social inequalities. It is important to notice that if we introduce the concept of an endogenous technological change, then the vagaries of technologies imply a set of possibilities even bigger. For example, with an endogenous technological change, it is possible to design models in which an exogenous increase in wages leads to a technical progress that contributes to make labor more productive (and, on the contrary, in which low wages contribute to maintain a technological equilibrium using low productive labor). See for instance Acemoglu 2010 and Assous-Dutt 2010. This type of models reinforces the possibility of self-sustained divergences and thus the legitimacy of a public intervention.

Appendix to Chapter 7. Inequality and Concentration: Preliminary Bearings

Tables 7.1, 7.2 and 7.3, replicated in the book on p.247-249, show the observed order of magnitude over time and space for labor income inequality, capital ownership inequality and total income inequality. Supplementary tables S7.1, S7.2 and S7.3 present which levels of income and wealth correspond to these distributions.

It is possible to modify these figures in the corresponding excel file. For instance, the average labor income per adult and per year of 24,000€ (2,000€ per month) is only given as an example and can be modified in table S7.1 in the excel file, which will automatically compute the average labor income of the different social groups with the new figures, on the basis of the distributions specified in table 7.1.

It also possible to change the distributions specified in tables 7.1-7.3 and then the incomes and wealth given in tables S7.1-S7.3 will automatically adjust. For instance, the wealth distribution presented as relatively equalitarian in table 7.2 (named “low inequality”) has in fact a pretty much inegalitarian (even if the observed distribution on the ground are always less equalitarian) and it is also feasible to imagine a more equalitarian distribution and to simulate the effect on the wealth of the different groups.

Note on the orders of magnitude specified in tables 7.1-7.3 (**p.250**)

I write in the book on p.250 that the rough estimates presented in these tables 7.1-7.3 correspond to distributions of incomes and wealth estimated within the adult

population at an individual level, and before taxes and transfers (they are thus primary distributions). Several things about this matter need to be pointed out. First, these estimates are approximations by construction: the aim of this chapter is to make to reader get used to the orders of magnitude that income or wealth inequality usually reach and not to analyze accurate statistics. Besides, as I write on p.250 (also see p.256 and p.269), there are several ways to take into account couples and individuals with no income, and these different methodological choices can have a significant impact on the orders of magnitude.

For instance, what are called replacement incomes (pensions and benefits), are usually integrated again in primary incomes (In a lot of countries, replacement incomes are based on contributory schemes: these incomes are often financed by contributions or taxes that are proportional to the previous wages, and are themselves almost proportional to the previous wages). Only "pure" transfers, as "revenu minimum vieillesse" in France ("old age minimum") (or other social minima) are excluded from primary incomes. Fiscal rules that apply in most of the countries generally follow the same convention (most of the replacement income are taxable whereas pure transfers are not). But drawing a clean line is not always obvious and this can sometimes bias the comparisons between countries. For example, in the United States, it is not clear how to treat public pensions scheme, as well as unemployment insurance. They are not really financed by wage deductions as in contributory schemes but could be considered more as pure transfers, as they are only partly taxable and taken into account in the declaration of incomes. As a consequence, if we try to use this fiscal data and this concept of fiscal income to measure the share of the lowest 50% incomes, we find a very small figure (11.9% of the total income in 2010)[27]. Including transfer payments would give share that is appreciably higher, about 17%-18% of the total income.[28] The estimate given in Table 5.3 (a share of 20% for the bottom 50% incomes) is certainly a bit too high; and it is likely that the estimate for 2030 of 15% is not far from already being reached.

The different ways to take into account couples also have an important impact. For instance, table B5 of this excel file present the estimates of the inequality in the American labor distribution, calculate either at the couple level (adding the wages of the two spouses, the same way the American tax is calculated), or at the individual

[27] See this table.
[28] However, the fact that it will not necessarily lead to a reduction of the top decile share has to be highlighted. Indeed, it would be necessary to add non-taxable capital income, which reached very high level during the 2000s-2010s to such an extent that this second effect could outreach the first one. See table 3 in this article.

level (considering each spouse as a different individual). We observe that moving from the couple level to the individual level slightly increases the top decile share in total wages (rising from 35% to 37% in 2000). This means that positive assortative matching (the fact that high wage people tend to get together) is high but lower than 100 %, and thus does not exactly compensate the fact that there are also single individuals, which increase the income inequality measured at the couple level. On the contrary, the share in total wages of the bottom half is lower when we take the individual level. Besides, the share of the bottom half varies a lot with economic cycles and the unemployment rate: it rises to 17% with full employment (around 2000) and falls down to 13% with under-employment (2010-2012).[29] Taking into account replacement incomes (specially unemployment benefits that can supplement the lowest wages for people who do not work the whole year), implies that the share of the bottom half in total labor income (replacement incomes included) is about almost 20%. The rough level of 25% mentioned in table 5.1 for the United Sates in 2010 is again certainly a bit too optimistic.

Note on wealth inequality by age groups

I write in the book on p.245 and p.258 that the concentration of wealth is nearly as great within each age cohort as it is within the population as a whole. For evidence on this, see for instance this paper, where we compare estimates of wealth concentration at death and among the living in France. See also the references given in this paper. In particular, see the UK estimates for top wealth shares by age group and for the population as a whole given by T. Atkinson, The economics of inequality, 1983, p.176, table 7.4.

Note on the calculations of the Gini coefficients (**p.266-267**)

I present on p.266-267 the notion of Gini coefficient, and I analyze the coefficients I got for the distributions specified in tables 7.1, 7.2 and 7.3. The detailed calculations for those coefficients are given in tables S7.4-S7.6. The exact formulas I used are available in this excel file. Examples of Gini-Lorenz's curves are available in figure S7.1 (which uses the series of table S7.7).

Figure S7.1. Examples of Gini-Lorenz curves

[29] These estimates were calculated based on a reference population which corresponds to the level of unemployment in 2000. See table B5.

It is useful to briefly recall some statistical notions to get these formulas. The Gini coefficient is a synthetic index to measure inequality ranging between zero (total equality) and one (total inequality). Technically, the Gini coefficient is defined as twice the area bounded by the distribution curve – also called the Gini-Lorenz's curve- and the 45° degree line. This is shown on figure S7.1 through two examples: one with a distribution with only two social groups supposed to be homogenous (the poorest 90 % and the richest 10%) and one with a continuous distribution. By convention, the y% share of the total income (or total wealth) owned by the poorest x% of the population is indicated on the vertical axis. If the distribution were perfectly equalitarian, the curve of such a distribution would be the 45° degree line and then the area between those two would be zero. On the contrary, the area is maximum when a minute percentage of the population owns 100% of the total, then the area equals ½ (hence the multiplication by two in order to normalize to one the Gini coefficient)[30].

In practical terms, the distributions observed in the real world are always continuous. However, it is useful to calculate Gini coefficients corresponding to a finite number of social groups. In order to do this, let p_1,..., p_n be the studied percentile and s_0,s_1,..., s_n be the corresponding share of the total income (or total wealth)[31]. s_0 is the share held by the people who are below the percentile p_1, s_1 is the share held by the people who are between the percentiles p_1 and p_2, ..., s_n is the share held by the people who above the percentile p_n. By definition, the sum of these shares is always equal to one: $\Sigma_{0\leq i\leq n}\, s_i = 1$.

[30] Founding articles are M. Lorenz's ones ("Methods of measuring the concentration of wealth", Journal of the American Statistical Association, 1905) and C. Gini ("Measurement of inequality of income", Economic Journal, 1921).
[31] In what follows, I do not systematically repeat « income or wealth », but it is obvious that these formulas apply for both of them.

Example 1. Let's assume n=1, p_1=0,9, s_0=0,5, s_1=0,5. This is a two-group distribution: the poorest 90%, holding 50% of the total income, and the richest 10%, holding 50% of the total income as well.

Example 2. Let's assume n=2, p_1=0,5, p_2=0,9, s_0=0,2, s_1=0,3, s_2=0,5. This is a three-group distribution: the poorest 50%, holding 20% of the total income, the following 40%, holding 30% of the total income, and the richest 10%, holding 50% of the total income.

Example 3. Let's assume n=3, p_1=0,5, p_2=0,9, p_3=0,99, s_0=0,2, s_1=0,3, s_2=0,3, s_2=0,2. This is a four-group distribution: the poorest 50%, holding 20% of the total income, the following 40%, holding 30% of the total income, the following 9%, holding 30% of the total income and the richest 1%, holding 20% of the total income.

We can prove that the Gini coefficient is given by the following formulas:

Two group case. If we calculate the area of the two triangles and the rectangle (cf. figure S7.1), we get: $G = 1 - p_1s_0 - 2(1-p_1)s_0 - (1-p_1)s_1$

Or (replacing s_0 by $1-s_1$) : $\mathbf{G = s_1 + p_1 - 1}$

For example, if the top decile owns 20% of the total, then G=0,2+0,9-1=0,1.
If it owns 50% of the total, then G=0,5+0,9-1=0,4.
If it owns 90% of the total, then G=0,9+0,9-1=0,8.

We immediately see how the Gini coefficient becomes close to zero with the most equalitarian distribution of labor income (Scandinavian type) and close to 1 with the least equalitarian distribution of labor income (Belle Époque type).

If $s_1 = 1 - p_1$ (the higher group owns exactly its share of the population), then, by definition we are in the perfect equality case: G = 0.
If $p_1 \rightarrow 1$ et $s_1 \rightarrow 1$ (the higher group is infinitely small and owns almost everything) then, by definition, we are getting closer to the total inequality case: $G \rightarrow 1$.

Three group case. Multiplying small triangles, this formula can be extended to cases with more than two groups (Brown formula). With three groups, we get: $G = 1 - p_1s_0 - (p_2-p_1)(2s_0 + s_1) - (1-p_2)(1+s_0+s_1)$

With four groups:

$G = 1 - p_1 s_0 - (p_2 - p_1)(2s_0 + s_1) - (p_3 - p_2)(2s_0 + 2s_1 + s_2) - (1 - p_3)(1 + s_0 + s_1 + s_2)$

With n+1 groups:

$G = G = 1 - p_1 s_0 - [\ \Sigma_{1 \le i \le n-1} (p_{i+1} - p_i)(2s_0 + 2s_1 + \ldots + 2s_{i-1} + s_i)\ ] - (1 - p_n)(1 + s_0 + \ldots + s_{n-1})$

Formulas application. I present in table S7.4-S7.6 the corresponding Gini coefficients for the two-group, three-group and four-group distributions. Unsurprisingly, the more we take into account different social groups, the higher is the index of inequality. With these calculations, it is easier to get used to the type of distribution leading to a particular Gini coefficient. In tables 7.1-7.3, I replicate the Gini coefficients corresponding to the four-group distribution (the one which is the closest to the reality). If the real continuous distributions that we observe were taken into account, the coefficient would be even higher. The recent formula proposed by Alvaredo 2011 clarifies the link between the Gini coefficients and the share of the top decile, centile, thousandth.

Note on the "social tables" of the 18th-19th centuries (pp.268-270)

The detailed references to first national accounts set up in different countries (in particular the United Kingdom, France and the United States) are given in Capital is Back..., 2013 (in particular appendix for each country).

Also refer to the following interesting documents that a good start with the “social tables” set up in the different countries:

- P. Lindert, J. Williamson, "Revising England's Social Tables, 1688-1812", Explorations in Economic History, 1982; "Reinterpreting Britain's Social Tables, 1688-1913", Explorations in Economic History, 1983. (U.K.)
- C. Morrisson, W. Snyder, "The Income Inequality of France in Historical Perspective", European Review of Economic History, 2000. (France)
- P. Lindert, J. Williamson, "American Incomes Before and After the Revolution", Working Paper 2011 (U.S.A.)
- B. Milanovic, J. Williamson, P. Lindert, "Measuring Ancient Inequality", Working Paper 2007 (This is certainly the most ambitious work, since the authors gather “social tables” set up since the Antiquity; the comparability between the data is however rather weak)

Appendix to chapter 8. Two Worlds

Series used to construct figures 8.1-8.2, replicated in the book on p.272-273, are available in table S8.1, as well as in the corresponding excel file.

Tableau S8.1. La part des hauts revenus et des hauts salaires en France, 1900-2010
(series used for figures 8.1-8.2)

These series, as well as most of the series presented in chapter 8 and 9, have been taken from the "World Top Incomes Database" (WTID). The series about France were initially set up and published in my book *Les hauts revenus en France au 20e siècle…*, Grasset 2001 (the main technical references used in this research and the corresponding links are presented above; see in particular the technical appendix of my 2001 book, which includes all the methodological details for this estimates). The series used here were completed with the data regarding the years before the creation of the income tax (1914 in France, first data on incomes appeared in 1915) using different estimates made in France during the Belle Epoque, reviewed and corrected on the basis of the first fiscal data. There were completed for the recent years from Landais 2007 's and Godechot 2012 's works and recent fiscal data. All the details regarding this update are available in the corresponding excel file

The series used to construct figures 8.3-8.4 and 8.9-8.10, replicated in the book on p.276-277 and 301-302, as well as the supplementary figures S8.1-S8.2 (which shows the same curves, without capital gains) are available in table S8.3, as well as in the corresponding excel file.

Table S8.3. Top income and top wage shares in the United States and in France
(series used for figures 8.3-8.4, 8.9-8.10 et S8.1-S8.2)

These data about the composition of high incomes have been taken from my book *Les hauts revenus en France au 20e siècle...*, 2001, for France (see detailed data here), and from the article "Income Inequality in the United States, 1913-1998", QJE 2003 (written with E. Saez), for the United States (see updated series here). All the details are available in this excel file. These data on the composition of high incomes are not available on the WTID website, because they are only available for few countries to date. For France and the United States, they are available on an annual, or almost annual, basis since the creation of the income tax, which allows more subtle analysis on the observed evolutions.

Note on the estimates of the level and the threshold of the top fractiles (**p.281-284**)

I present in the book on p.281-284 some limitations linked to the series on the top fractiles of high incomes presented hereby. For further precisions, I invite the interested reader to refer to the WTID, to my book *Les hauts revenus en France...*, 2001, and to the two books *Top Incomes Over the 20th Century...*, Oxford University Press, 2007, and *Top Incomes: A Global Perspective*, Oxford University Press, written with A. Atkinson (refer to the main technical references of which links are given at the beginning of this appendix).

It has to be mentioned that the data given here only refers to average incomes above the threshold of the top fractiles (average income for the top decile, the top centile…), and not to the threshold that has to be reached to enter these groups. Complete series on both the average incomes and the threshold of the top fractiles are available on the WTID and in the chapters specific to countries in the two volumes published in 2007 and 2010. The different chapters also try to evaluate the bias of the estimates due to the different tax rebates, especially for the capital incomes.

One more thing should be brought to the fore. As explained in the book on p.281-284, the estimates currently available in the WTID do not propose a systematic correction for this bias, and for this reason, probably somewhat under-estimate the level and the rise of income inequality. Thanks to several sources, it is possible to take into account the capital incomes that do not appear in the declaration of incomes (the reason may or may not be legal) and thus to correct the top decile shares. We can for instance use the estimate of the financial assets held in tax

havens (in particular, refer to Zucman's estimates quoted in chapter 12 of the book). If the missing financial assets account for 10 % of the national income, the missing annual flow can reach 0.5% of the national income. We can also use the estimates of the wealth distribution. For example, during the Belle Epoque, around 90% of the wealth was owned by the top 10%, of which more than 50% by the top 1%. Given that the capital share reaches 35-40% of the national income, we can estimate that the shares of the top 10% and top 1% in the income distribution are closer to 50% and 20-25% than 45% and 20%. A more systematic attempt to correct these estimates looking at national accounts was proposed by Landais-Piketty-Saez 2011. The idea is to recalibrate the set of individual incomes in order to replicate the mass of dividends, rents, etc…, observed in the national accounts. One of the main limitations of this method is the assumption of proportional allocation.

Figure 8.5. Income inequality in the United States, 1910-2010 (**p.291**)
Figure 8.6. Decomposition of the top decile, United States, 1910-2010 (**p.292**)
Figure 8.7. High incomes and high wages in the United States, 1910-2010 (**p.299)**
Figure 8.8. The transformation of the top 1 percent in the United States (**p.300**)

The series used to construct figures 8.5-8.8, replicated in the book on p.291-300 are available in table S8.2, as well as in the corresponding excel file excel file.

Table S8.2. Top income and top wage shares in the United States, 1900-2010
(series used for figures 8.5-8.8)

The series used were initially set up and published in the article "Income Inequality in the United States, 1913-1998", QJE 2003 (written wih E. Saez). See updated series and the WTID website. The series used here were completed with the data for the years before the creation of the income tax (1913 in the United States, first data on incomes appeared in 1913) using different estimates made in the United States around 1900-1910, reviewed and corrected on the basis of the first fiscal data.

It is worth paying attention to estimate of the American incomes distribution around 1910 made by King 1915. More precisely, King gives in his book p.224-229 a very detailed distribution for the United States, based in particular on the estimate made by the American tax administration and the Congress in the foreseeing of the creation of the income tax; King finds around 38% for the top 10% and 15% for the top 1%, which seems a bit too low given the first fiscal statistics (we notice the same kind of under-estimation in France around 1900-1910). As I explain in the book, King is

especially interested in the comparison between the United States and countries with more inequality – at that time - as Prussia, France and the United Kingdom. For other comparisons of inequalities in the New World and in Europe around the 1910s, also refer to Procopovitch 1926. Also see *Les hauts revenus en France...*, 2001, p.485-486.

Appendix to chapter 9. Inequality of Labor Income

The series used to construct figure 9.1, replicated in the book on p.309, as well as the supplementary figures S9.1-S9.2, are available in table S9.1, as well as in the corresponding excel file.

Table S9.1. Hourly minimum wage in France or in the USA, 1950-2013
(series used for figures 9.1 et S9.1-S9.2)

The series presented in this table have been taken from the social legislation in these two countries (see the excel file for the details).

The series used to construct figure 9.2-9.9, replicated in the book on p.316-327, as well as the supplementary figures S9.3-S9.6, are available in tables S9.1-S9.5, as well as in the corresponding excel file (also see the excel file of chapter 8).

Table S9.4. Share of the top incomes in the total revenue: Europe and the USA, 1900-2010
Table S9.5. Share of the top incomes in the total revenue: India, South Africa, Indonesia, Argentina, China, Colombia, 1900-2010
(series used for figures 9.2-9.9, S9.3-S9.5)

All the series have been taken from the "World Top Incomes Database" (WTID), with a number of corrections and extensions. They are described in a detailed manner in this corresponding excel file (as well as the excel file of chapter 8).

It is impossible to give here the complete list of authors and sources used to construct to all these series. For further precisions, I invite the interested reader to refer to the WTID website, and especially to the chapters specific to countries in these two books *Top Incomes Over the 20th Century...*, Oxford University Press, 2007, and *Top Incomes: A Global Perspective*, Oxford University Press, written with A. Atkinson (refer to the main technical references of which links are given at the beginning of this appendix). These chapters give all the details required to understand the particularities of the evolutions specific to each country.

Among the corrections more precisely described in the excel files of chapter 8 and chapter 9, the following ones can be specified. In Sweden, in 1903 and in 1906, the raw data show top centile shares reaching 27-28%. It may be due to purely statistical problems (in particular the data for 1903 is not fully coherent). We then replace the values for these years by moving average around these years (hence a top centile share around 23%-24%). The same phenomenon occured for Denmark in 1915-1918.

In Germany, series are highly unstable, which is due to both the frequent political, territorial and institutional changes and to the fact that fiscal data are generally not available every year. The conclusion reached by Fabien Dell, that the share of high incomes in Germany is bigger than in other continental European countries, seems robust. See Bach-Corneo-Steiner 2011, who find a top 0.1% share about 4% of the total income (versus barely 2% in France, in Japan or in Sweden)

The data for India and China for the recent period have to be taken very carefully. The publications of fiscal data we used with A. Banerjee for the period 1922-2000 (see here) were interrupted by the Indian administration. I extended them using an

average estimate of different “executive pay" - type surveys.[32] These data have to be taken extremely carefully and are only indicative. The Chinese data are even weaker. The series built with N. Qian are based on data from surveys and severely under-estimate the highest incomes (see here). I thus add a 40% increase in these series, which seems plausible given the under-estimation of the Chinese income tax revenues. Yet, I would like to point out the fact that these revenues are perhaps themselves lower than they should be. It is thus not excluded that these series under-estimate the current income inequality in China.

[32] For example "Top executive compensation report 2011-12", Hay Group, 2013.

Appendix to chapter 10. Inequality of Capital Ownership

The series used to construct figures 10.1-10.6, replicated in the book on p.340-348 are available in table S10.1, as well as in the corresponding excel file.

Table S10.1. The concentration of wealth in Europe and in the USA, 1810-2010
(series used for figures 10.1-10.6)

All the details on the sources and the methods used to construct these series are in the excel file. Only the main points are presented here. First, the estimates for the wealth inequality are calculated among the adult population, which always leads to higher inequality than if it was measured at deaths (using "mortality multiplier" techniques enable to switch from one to the other, this is explained in detail by Kopzuck-Saez 2004 and Piketty-Postel-Vinay-Rosenthal 2006).

A very interesting general reference on the historical evolution of the concentration of wealth is the book Waldenstrom 2009, Lifting all boats… In particular table 3.A1 p.120-121 (Sweden) and tables 4.A1-A3 pp.148-154 (Denmark, France, Norway, Switzerland, UK, US). These are very complete tables gathering series on the concentration of wealth in a large number of countries. Also see Ohlsonn-Roine-Waldenstrom 2008 and Waldenstrom 2012. On the United States and the United Kingdom, also see Lindert 2000 (Handbook of Income Distribution), in particular table 2 pp.181-182 and table 3 p.188. For the United Kingdom, Lindert relies specially on its article JPE 1986. The survey Lindert 2000 is however somewhat outdated and less complete than Waldenstrom 2009. On Australia, see the recent study available here.

The main sources used to construct the series presented in table S10.1 are the following ones (excel file):

France. Calculations are based on the series from Piketty-Postel-Vinay-Rosenthal 2006 for 1810-1990 (built from inheritance data), completed by the estimates of Landais-Piketty-Saez 2011 for 2010 – that are built with the combination of different types of data, in particular inheritance data and fiscal data from the ISF (Impôt de Solidarité sur la Fortune, the French wealth tax). All series were homogenized in order to relate to the wealth distribution of the livings. The huge uncertainties on the more recent estimates, which certainly under-estimate the wealth of the wealthiest, needs to be pointed out. In particular, the self-reported surveys on wealth done by INSEE, have to be taken carefully because they under-estimate a lot the top of the distribution in comparison to fiscal data (which are themselves a lower bound for the wealthiest). For instance, the surveys done by INSEE in 2004 and 2010 give a top decile share just above 50% of the total wealth (see here), whereas fiscal data (inheritance and ISF) suggest a top decile share above 60% of the total wealth, which, besides, is more coherent with the results for other countries.

Sweden. See Waldenstrom 2009, Table 3.A1, p.120-121.

United Kingdom. See Atkinson et al 1989 Table 1 for 1923-1981. Atkinson et al 1989 are the extension of Atkinson-Harrison 1978; the series only start in 1923 given the absence of cross data for the size of inherited wealth and age before this year. However, Atkinson-Harrison 1978 table 6.1 give an estimate of the top 1% share for 1911-1913 on the basis on the available imperfect data (they get 69% of the wealth for the top 1% to be compared with 61% in 1923). These series are also used in Waldenstrom 2009 pp.148-154. I completed these series with the IRS series for 1980-2010, and the Lindert's estimates for 1810-1910. See excel file.

United States. Totally homogenous and satisfactory historical series do not exist for the United States (this can be partly explained by the fact that the federal inheritance tax was created only in 1916 and has always concerned only a small part of the population). The series given here for the period 1989-2010 are based on estimates from the Survey of Consumer Finances: see Kennickell 2009 Table 4; Kennickell et al 2011 Tables 2-3; Wolff 2010 Table 2);[33] for the period 1962-1989 in Wolff 1994; and for the period 1916-1962 on the estimates of Kopczuk-Saez 2004 Table B1 (based on inheritance data) homogenized with later series (see this fichier excel for the details). These different series are also used in Waldenstrom 2009 pp.148-154, who however does not attempt to homogenize the raw estimates. For period 1810-1910, I

[33] Pour des tentatives d'amélioration de la fiabilité des estimations SCF pour le haut de la distribution pour la période récente, voir Johnson-Shreiber 2006 et Raub-Johnson-Newcomb 2010.

use the data from the Lindert 2000's survey (estimate for the total population, including slaves). Huge uncertainties exist on these estimates. See Gallman 1969's estimates, who got wealth concentrations for the United States higher than Lindert (around 60-70% for the top decile). Also see L. Soltow, *Distribution of wealth and income in the United States in 1798*, University of Pittsburgh Press, 1989

Examples of the distribution of lands in traditional societies (**p.345**)

I write in the book on p.345, that the distribution of farmland in traditional rural societies is characterized by a very high concentration, typically with 80%-90% of the total of by the top decile. I give the example of the article of R. S. Bagnall, Journal of Roman Studies, 1992, which gives an estimate of the distribution of lands for Egypt under the Roman Empire. For an estimate on the 1950s, and according to which around 80% of the farmlands were held by the top decile, refer to P. Verme "Inside Inequality in Egypt: Historical trends, recent facts, people's perceptions and the spatial dimension", World Bank 2012, table 2, p.7.

Figure 10.7. Return to capital and growth: France 1820-1913 **(p.352)**
Figure 10.8. Capital share and saving rate: France 1820-1913 **(p.352)**

The series used to construct figures 10.7-10.8, replicated in the book on p.352, are available in table S10.2, as well as in the corresponding excel file.

Table S10.2. Capital rate of return, growth rate, capital share and savings rate in France, 1820-1910 *(series used for figures 10.7-10.8)*

This table has been taken from On the Long-Run Evolution of Inheritance..., 2010 (in particular Appendix A). For interested readers, see this research or its summarized version (QJE 2011).

Figure 10.9. Rate of return versus growth rate at the world level, from Antiquity until 2100 (**p.354**)
Figure 10.10. After tax rate of return versus growth rate at the world level from Antiquity until 2100 (**p.356**)
Figure 10.11. After tax rate of return versus growth rate at the world level from Antiquity until 2200 (**p.357**)

The series used to construct figures 10.9-10.11, replicated in the book on p.354-357, are available in table S10.3, as well as in the corresponding excel file.

Table S10.3. Capital rate of return, growth rate for the world, 0-2200
(series used for figures 10.9-10.11)

The assumptions on which these series are based are described in the book on p.353-358. All the details are available in this excel file.

Note on the “modified golden rule": $r = \theta + \gamma\, g$ (**p.360**)

I write on p.360 that the equilibrium rate of return to capital, in the framework of a infinite-horizon economic model, is given by the “modified golden rule" formula:

$$r = \theta + \gamma\, g$$

More precisely, it can be proved that this conclusion immediately stems from the hypothesis (even unlikely) according to which a unique representative agent correctly describes the behavior of the economic agents. Namely, in this model, everyone maximizes an infinite-horizon utility function with the following form:

$$U = \int_{0 \leq t \leq +\infty} e^{-\theta t}\, u(c_t)$$

In which θ is the time preference rate, $u(c) = c^{1-\gamma}/(1-\gamma)$ the utility function for consumption, and γ measures the concavity of the utility function (this parameter is generally assumed to be higher than 1 and usually ranging from 1,5 to 2,5).

Intuitively, $r = \theta + \gamma\, g$ is the unique rate of return to capital possible in the long-run for the following reason: the is the sole rate such that the agents are willing to rise their consumption at rate g, that is at the growth rate of the economy. If the return is higher, the agents will prefer to postpone their consumption and accumulate more capital, which will decrease the rate of return; and if it is lower, they will want to anticipate their consumption and borrow more, which will increase the rate of return.

For mathematical details, refer for instance to these lecture notes. Formally, it should be noted that the model requires that the “transversality condition” ($r>g$) holds (even if $\gamma<1$). This is required to make sure that the discounted present value of the future incomes is bounded.

Note on the property division regime brought in by the Civil Code (**p.576**)

For a more detailed description of the rules for property division and the new marital property regime established by the Civil Code, for example refer to T. Piketty, G. Postel-Vinay et J.L. Rosenthal, « Inherited vs Self-Made Wealth: Theory and Evidence from a Rentier Society (Paris 1872-1937) », *Explorations in Economic History*, 2013, long version. Also see the works of N. Frémeaux et M. Leturcq, « Régimes matrimoniaux et contrats de mariage en France depuis la Révolution », PSE, 2013 (available in N. Frémeaux's thesis, online here).

Note on the link between the Pareto coefficient and the gap r - g (**p.364-368**)

I write in the book on p.364-368 (and p.373-375) that the dynamic models of wealth accumulation based on multiplicative shocks generate Pareto distributions, and that the coefficient measuring the inequality of these Pareto distributions is an increasing function of the gap r - g (for a given structure of shocks).

More precisely, Pareto distribution is a particular form of distribution that follows this kind of mathematic law:

$$1-F(y)=(c/y)^{a}$$

In which 1-F(y) is the share of the population whose income or wealth is higher than y, c is a constant and a the coefficient of the Pareto law.

The idiosyncrasy of the Pareto laws is that if we calculate the average income or wealth y* of all the people whose income or wealth is higher than y, then the y*/y ration is equal to a constant b. This coefficient, called "inverted Pareto coefficient", is simply linked to the coefficient:

$$b = a/(a-1)$$

Intuitively, the higher is b, the thicker is the top of the distribution, and thus the stronger is the concentration of wealth. As a consequence, the coefficient b measures the distribution inequality (whereas the coefficient a varies in the opposite direction, and thus measures the distribution equality). On the mathematical link

between a and b, and the way they can be empirically measured, see for instance, L*es hauts revenus en France au 20e siècle…*, 2001, Annexe B.

Pareto laws are generated by dynamic process with multiplicative shocks, for instance because portfolios and wealth are multiplied by a random multiplicative shock from a period or a generation to another. Intuitively, the bigger the gap r – g is, the more this shock generates a high concentration of wealth and thus the higher the coefficient b is. The mathematical equations to determine this coefficient b depending on r – g, as well as the corresponding technical references are available in Piketty-Zucman, "Wealth and Inheritance in the Long Run", 2014. See article in pdf format p.42-47. See also these lecture notes. The corresponding models were developed by Champernowne in 1953 and extended by several authors, such as Stiglitz, Cowell and Nirei. Meade developed a similar intuition in his book published in 1964[34]. What needs to be emphasized is that small variations in the gap r-g (due to differences in the tax rate for the wealthiest for example) can explained huge variations in the coefficient b and thus in the concentration of wealth. For numerical simulations see Dell 2005. See also the numerical simulations presented in Piketty-Zucman, "Wealth and Inheritance in the Long Run", p.42-47.

In the real life, inverted Pareto coefficient b typically range from barely 1.5 (low inequality) to 3-3.5 (very high inequality). To consult charts representing the evolution of the Pareto coefficients for the income distribution in the different countries by the WTID over the past century, see Atkinson-Piketty-Saez, "Top Incomes in the Long Run of History", *Journal of Economic Literature*, 2011 (in particular figures 12-15 p.50-55 on the evolution of coefficient b and table 3 p.14 for the link between a et b).

The Pareto coefficients link in a simple manner the shares of the top decile, centile, thousandth and so on (discussion in the book on p.367-368). In concrete terms, the share of the population above y is equal to $1-F(y)=(c/y)^a$. Then, for 2 percentiles p<q (for instance, p=0,1%, q=1%), we have $p=(c/y_p)^a$ and $q=(c/y_q)^a$, hence $y_p/y_q = (q/p)^{1/a}$ (in which y_p et y_q are the thresholds corresponding to the percentiles p and q). For q/p=10 (for example we want to know the top 0.1% into the top 1% or the top 0.01% into the top 0.1%, etc.), we get the following orders of magnitude:

- if a=3 (b=1,5, low inequality), then $y_p/y_q = 10^{1/3} = 2,15$

[34] Meade does not go as far as studying the Parto equilibrium law, but he insists on the fact the dynamic of inequality becomes more and more explosive when the "internal rate of reproduction" (the product of the savings rate with the rate of return) of the largest wealth is higher compared to the growth rate.

- if a=2 (b=2, medium inequality), then $y_p/y_q = 10^{1/2} = 3,16$
- if a=1,5 (b=3, high inequality), then $y_p/y_q = 10^{1/1,5} = 4,64$
- if a=1,4 (b=3,5, very high inequality), then $y_p/y_q = 10^{1/1,4} = 5,18$

Given that the average income or wealth above the threshold are proportional to the threshold, it stems from it that a group 10 times smaller owns (as a share of the total) 20% of what a group 10 times bigger owns in a distribution with low inequality, 30% in a distribution with medium inequality, and more than 50% with very high inequality.

Note on the end of the hyper-inegalitarian equilibrium (**p.368-370**)

I mention on p.368-370 the end of the hyper-inegalitarian "rentier" equilibrium is due to the shocks of World War I. While the heirs of the period 1872-1912 managed to leave enough wealth as inheritance to finance the standard of living they experienced themselves, this equilibrium broke between the two wars. On this specific point, see « Inherited vs Self-Made Wealth...», 2013 (in particular long version, figure 12).

Table 10.1. The composition of Parisian portfolios, 1872-1912 (**p.371**)
Table S10.4. The composition of Parisians portfolios in 1872-1912

Table 10.1, replicated in the book on p.371, of which a detailed version (with the decomposition of the different foreign assets according to the level of wealth) is available in the supplementary table S10.4, have been taken from « Inherited vs Self-Made Wealth...», 2013 (table 4 and technical appendix, table B11). All the detailed calculations are given in the corresponding excel file.

Appendix to Chapter 11. Merit and Inheritance in the Long Run

The series used to construct figures 11.1-11.11, replicated in the book on p.380-421, are available in tables S11.1a, S11.1b et S11.2a, as well as in the corresponding excel file.

Table 11.1. The age-wealth profile in France, 1820-2010 (**p.394**)
Table S11.1a. The inheritance flow in France 1820-2100
Table S11.1b. Inheritance and labor for cohorts born in the 1790s-2030s
Table S11.2a. Share of inherited wealth in total wealth in France, 1850-2100
(series used for figures 11.1-11.11)

These tables have been taken from *On the Long-Run Evolution of Inheritance...*, 2010. All the details on the sources and the methods used in this research are available online (long version or short version), as well as the annual series corresponding the decennial averages presented here.

The following supplementary figures present different variants of figure 11.7.

Figure S11.1. Share of inherited wealth in total wealth, France 1850-2100 (1)
Figure S11.2. Share of inherited wealth in total wealth, France 1850-2100 (2)

Figure S11.3. Share of inherited wealth in total wealth, France 1850-2100 (3)
Figure S11.4. Share of inherited wealth in total wealth, France 1850-2100 (4)
Figure S11.5. Share of inherited wealth in total wealth, France 1850-2100 (5)
Figure S11.6. Share of inherited wealth in total wealth, France 1850-2100 (6)
Figure S11.7. Share of inherited wealth in total wealth, Paris 1872-1937 (1)
Figure S11.8. Share of inherited wealth in total wealth, Paris 1872-1937 (2)
Table S11.2b. Share of inherited wealth in total wealth in Paris, 1872-1937
(series used for figures S11.7-S11.8)

All the details are given in the corresponding excel file. Let me simply state here the following points. Several definitions of the cumulated share of the inherited wealth in the total wealth exist. In the definition proposed by Modigliani, past inherited wealth flows are not capitalized at all. In the definition proposed by Kotlikoff-Summer, they are entirely capitalized using the average rate of return. None of them is really satisfactory. On this issue, see On the Long-Run Evolution of Inheritance..., 2010, p.72-77. A more satisfactory definition would be to explicitly acknowledge that two distinct groups always exist in the population (the ones whose wealth is smaller than the capitalized value of their inherited wealth, and the ones whose wealth is larger than the capitalized value of their inherited wealth), and, for the first group, to set the capitalized value of the inherited wealth equal to the current wealth only. Unfortunately this definition requires more data, which is another difficulty. This definition is proposed and applied to the very extensive Parisian inheritance dataset in "Inherited vs Self-Made Wealth: Theory and Evidence from a Rentier Society (Paris 1872-1937)", *Explorations in Economic History*, 2013 (article written with G. Postel-Vinay and Jean-Laurent Rosenthal; see the long version, 2011). The data used for figures S11.7-S11.8 and table S11.2b have been directly taken from this research. The estimate for France mentioned on figure 11.7, supplementary figures S11.1-S11.6 and table S11.2a are rough estimates in the spirit of this research. All the details are given in the corresponding excel file[35]

[35] In concrete terms, I apply ratio closed to the ones I got in Paris in 1872-1937 to switch from to Modigliani definition to the PPVR definition. However the difference between Paris and the rest of France needs to be pointed out: there were both more inequality and more mobility in Paris in the 19th century, simply because the population had grown much more and also because it renewed faster. W can also apply the following method: without direct information on the joint distribution $F(b_{it},w_{it})$ between the inherited wealth and the actual current wealth, we can get a lower bound for the share of the inherited wealth, assuming no correlation between the labor income y_{lit} and the received bequests b_{it} (with the savings and return rates as well, s_{it}, r_{it}). Similar calculations show that it seems to give some results very close to these ones. The method based on separate assets/community assets used for Paris 1872-1937 can also be used here with the files DMTG 1984-2000. Some preliminary calculations give as well some results that are coherent with these ones.

The following supplementary figures present different variants of figures 11.9-11.11:

Figure S11.9. The share of inheritance in the total resources (inheritance and work) of cohorts born in 1790-2030
Figure S11.10. The dilemma of Rastignac
Figure S11.11. Which fraction of a cohort receives in inheritance the equivalent of a lifetime labor income?

The difference with figures 9.9-9.11 is that I represent on figures S11.9-S11.11 not only the results from the simulations of the central scenario (based on the hypothesis g=1,7% and r=3,0% for the period 2010-2100), but also the results for the alternative scenario (with g=1,0% and r=5,0% for the period 2010-2100). These different scenarios, as well as a lot of other variants, are analyzed in details in *On the Long-Run Evolution of Inheritance...*, 2010. For simulations corresponding to the dilemma of Rastignac, estimated at the 10% and the top 0.1% level (and not only the top 1%), see also appendix D, table D7.

Note on the share of annuitized wealth in total wealth (**p.390-393**)

I write on p.390-393 that the share of annuitized wealth, which cannot be passed on to descendants, account for less than 5% of private wealth in France, and can reach 15-20% in countries where pension funds are the most developed. For the detailed data on France, mentioning that the share of annuitized wealth is currently around 3% of wealth, refer to *On the Long-Run Evolution of Inheritance...*, 2010, Appendix A, pp.37-39. In the wealth accounts, the assets classified as "life insurance and pension funds assets" can account for 30-35% of the total wealth (for instance, refer to Capital is Back..., 2013, Table UK.6d). But a large share of these assets is actually annuitized (for example, a half in the United States, see Kopczuk-Saez 2004). The data gathered by the OECD on pension funds show maximum capitalizations ranging from 70% to 90% of the GDP, thus around 15%-20% of the total private wealth (see this table, it is yet not certain that all the components that can be passed on were deducted from these estimate)

Note on the formula $\mu \times m = 1/H$ (**p.401**)

I write on p.401 that for a low growth, the product $\mu \times m$ tends to get closer to 1/H (in which H is the length of a generation, that is about 30 years), so that the annual inheritance flow b_y gets closer to β/H (or typically 20% of the national income if the

capital/income ratio β is equal to 600%). The intuition for this result is that the ratio μ gets closer to the ratio (D-A)/H when the growth slowdowns, in which D-A is the length of an adult life. Given that the adult mortality rate m is 1/(D-A) with a stationary population, it stems from it that the product μ x m gets closer to de 1/H, and thus does not depend on the life expectation. The proof of this result is given in *On the Long-Run Evolution of Inheritance...* (version QJE 2011, p.1105-1116).

Note on the conditions such that rentiers are predominant over managers (**p.411**)

Generally, the condition can be written $p_k \times \alpha^* > p_l \times (1-\alpha^*)$, in which p_k is the share of the percentile we consider (for instance the top centile) in the inherited capital distribution, p_l is the share of the percentile under consideration in the labor income distribution, and α^* is the share of the inherited wealth in the total resources. The parameters p_k et α^* (respectively measuring the concentration and the total amount of inheritance) are both given as increasing functions of the gap r – g, the gap between the net rate of return to capital and the growth rate, so that rentiers are even more likely to dominate the managers if r-g is high. For further details, refer to these lecture notes.

Note on the meritocratic extremism (**p.416**)

I point out on p.416 the importance of meritocratic beliefs in modern societies. The Beckerian human capital theory specially emphasis this belief: "I think we have become more meritocratic - educational attainment has become increasingly predictive of economic success" (interview de Gary Becker with Alexander Stille, in New York Times, October 22 2011, "The Paradox of the New Elite"). It is interesting to notice that this belief does not rely on any empirical measure proving a trend toward a greater intergenerational mobility of skills and labor incomes over time (discussed in chapter 12).

Note on the high-ranking civil servants in France during the 19th century (**p.417**)

The quote on p.417 has been taken form a report presented by Thiers on December 30, 1831 as reporter for the budget commission about expenditures. In *Archives parlementaires* (Madival-Laurent), volume 73, pp.304-319. The documents of the Parliament have extremely detailed information on the evolution of the salary scales in the public services since the French Revolution. See *Les hauts revenus en France...*, 2001.

Figure 11.12. The inheritance flow en Europe, 1900-2010 (**p.425**)

The series used to construct figure 11.12, replicated in the book on p.425 are available in table S11.3, as well as the corresponding excel file excel file.

Table S11.3. Inheritance flow in Europe, 1900-2010
(series used for figure 11.12)

Estimates for the United Kingdom are provisional and come from Atkinson 2012. Estimates for Germany are also provisional and come from Schinke 2012. All the details are given in the excel file.

Appendix to Chapter 12. Global Inequality of Wealth in the 21st Century

The series used to construct figures 12.1-12.3 and table 12.1, replicated in the book on p.433-435, are available in tables S12.1 et S12.3, as well as in the corresponding excel file.

These tables come from the data used for the billionaires rankings published by the magazine Forbes from 1987 to 2013. It should be noted that I deducted the inflation in dollars, which on the period 1987-2013 is a little bit higher than the inflation in euros (refer to chapter 1). Measured in euros, the growth rate of the very high wealth has been a bit higher. But given the very high rates observed, it does not really make any difference (this would slightly accentuate the results). All the details are available in the excel file.

Note on the global wealth reports (**p.436-439**)

Some of the "global wealth reports" published the past years by the international financial institutions, of which I summarize the content in the book on p.436-439, are listed in this directory. The comparison between the number of French and German billionaires proposed in the Credit Suisse report, of which I talk about in p.437, is biased given that, in order to estimate the wealth distribution, the report uses fiscal data in France (from this article) and self-reported data from surveys in Germany; thus the top of the distribution is under-estimated for Germany as compared to France; note also that this is contradictory with the fact that German fiscal data shows a higher concentration than in France (read here).[36]

[36] Eventually, the report presents a share of 29 % of the total wealth for the top 1% in France against only 17% in Germany (see CS Global Wealth Databook 2010 p.120), which is not realistic.

Note on the share of the top wealth fractiles in the world wealth (**p.436**)

The estimates given in the book on p.436 - almost 20% of the global wealth owned by the top thousandth, 50% owned by the top centile and between 80-90% owned by the top decile – are not precise estimates and must be thought as orders of magnitude. They are coherent with the estimates given in the Global Wealth Reports (GWR) of the Credit Suisse, which are surely the most sophisticated available estimates among the "global wealth reports" published by the financial institutions (however still rough). For example the GWR 2012 mentions a share of 39.3% for the wealthiest 0.6%, and 43.1% for the next wealthiest 7.5%, 22.5% for the next wealthiest 14,4%, and 3.3% for the poorest 69,3% (figure 1, p.18). It should be pointed out that if the figures from the GWR are closed to the ones I mention in the text, all my figures about wealth are higher. In particular, the GWR 2012 estimates the global private wealth around 230,000 billion dollars (current exchange rate), that is around 280,000 billion dollars in purchasing power parity, whereas my estimate is 340,000 billion dollars PPP (280,000 billion euros), thus a 20% gap. In other terms, the global wealth (measured in PPP) is closer to 50,000 euros per adult according to GWR, and to 60,000 euros according to my estimate. I think my estimate may better reflect the increase of the capital/ income over the past decades (in particular, I think that the GWR's estimate seems to rely too much on wealth surveys in a lot countries where balance sheets do not exist). But the uncertainties are such that the magnitude of the errors could be 10-20%. These problems should be addressed in future research.

These differences between the estimates of the global wealth also appear among the different fractiles. For example, GWR 2012 estimates that 29 million people own more than 1 million dollars, for a total wealth of 87.5 trillions dollars (that is more than 3 million euros each on average). But several of these estimates rely on data from surveys (Germany for instance, see above) and under-estimate a lot the level of the top decile and top centile. Eventually, we can consider that everyone in the top centile (44.5 million people) owns more than one million euros and on average 3 million euros (in PPP).

It should be noted that the estimates from the reports Cap Gemini/Merril Lynch are much lower since their definition for the wealth is narrower: the HNWI (high net worth individuals) are defined as the ones who own more than 1 million dollars of wealth "they can invest"; that is financial assets that are of interest to asset managers, which exclude real estate and wealth from own businesses (family firms). The figure they

got (around 11 million dollars) is not really comparable to the estimates of Credit Suisse. On the opposite, the estimates given by Henry 2012 are much higher (but certainly overstated, see below)

Table 12.2. The return on the capital endowments of US universities, 1980-2010 (**p.448**)
Table S12.2. The return on the capital endowments of U.S. universities, 1980-2010

The data presented in table 12.2, replicated in the book on p.448, and in supplementary table S12.2, have been taken from calculations based on data published by the US universities. In particular, see the financial report available in this directory. All the details on the calculations are in the corresponding excel file.

Note on the foundations: private interests and general interest (**p.452**)

I refer on p.452 to a research from Camille Landais and Gabrielle Fack dedicated to tax rebates for foundations, and the porous frontier between private interests and general interest. This research is available here.

According to the US national accounts, foundations and other “NPISH” hold around 6%-7% of the total private wealth (see chapter 5). Yet, it is difficult to perfectly describe this large body. The total endowments of universities is barely 400 billions dollars in 2010-2012, that is less than 1% of the private wealth. The big other foundations (Gates’ foundations, Mc Arthur, etc., listed on foundationcenter.org for instance) hold around the same amount of assets.

Note on the return on sovereign wealth funds (**p.456-458**)

Financial report published by the different funds, of which I summarize the content in the book on p.456-458, are available in this directory.

The data on the oil and mining rent are available here (the world resources, as a whole, currently yield 5% of the world GDP, of which half for oil). See forecasts here. Logically, the exploitation of natural resources corresponds to a loss of capital, and should not be considered as an income. The rent from natural resources should be entirely invested again to avoid a negative income (Hartwick’s rules, AER 1971).

Figure 12.4. The world capital/income ratio, 1870-2100 (**p.461**)
Figure 12.5. The distribution of world capital, 1870-2100 (**p.462**)
Table S12.4. Private capital in the world, 1870-2100 (estimate)
(series for figures 12.4-12.5)

The series used to construct figures 12.4 et 12.5 are available in table S12.4 and in the corresponding excel file. These estimates are based on assumptions already presented in chapter 5 and in the appendix to chapter 5. All these details are available in this excel file.

Figure 12.6. The net foreign asset position of rich countries, 1985-2010 (**p.466**)
Table S12.5. Net foreign assets held in rich countries and tax havens
(series used for figure 12.6)

The series used to construct figures 12.4 and 12.5 are available in table S12.4 and in the corresponding excel file. These estimates come from the article Zucman 2013 (Figure 1). All the details are given in this article. One of the main differences between Zucman 2013's estimate of the financial assets held by households in tax havens (around 6-7 trillion dollars) and the one proposed by Henry 2012 (up to 20-30 trillion dollars) or, to a limited extent, the in-between estimate by Palan-Murphy-Chavagneux 2010 (around 10-12 trillion dollars) is that Henry relies, mostly, on the size of the portfolios declared by asset managers. Yet, there is a reasonable ground to think that these institutions tend to overstate their global balance and that such a method leads to double counting. I would like to stress one more time the uncertainties regarding these estimates. To find an interesting attempt to correct the estimates of the wealth concentration in Sweden adding non-registered assets held abroad, see Waldenstrom 2012, slide 36.

Appendix to Chapter 13. A Social State for the 21st Century

Figure 13.1. Tax revenues in rich countries, 1870-2010 **(p.475)**
Table S13.1. Public spending in rich countries, 1870-2010
(series used for figure 13.1)

The series used to construct figure 13.1, replicated in the book on p.475, are available in table S13.1. All the details on the sources I used are given in the following excel file.

Note on the structure of public spending (**p.477-479**)

The figures mentioned in the book on p.477-479 on the structure of public spending in rich countries mainly come from the following table:

Table S13.2. Public spending in rich countries (average 2000-2010, % GDP)

This table has been taken from this article (table 1). All the details on the sources I used are given in the following excel file.

For more detailed comparison on the structure of public spending, refer to Adema, W., P. Fron and M. Ladaique, 2011. “Is the European Welfare State Really More Expensive? Indicators on Social Spending, 1980-2012; and a Manual to the OECD Social Expenditure Database”, OECD Social, Employment and Migration Working Papers, No. 124. This paper is available here.

Note on the American imprisoned population (**p.579)**

The figures mentioned in the book on p.579 regarding the US imprisoned population come from Federal Bureau of Prisons.

Note on tax revenues in Europe (**p.481**)

The figures mentioned on p.481 on tax revenues in Northern Europe and Eastern Europe come from the Eurostat annual reports.

Note on the intergenerational mobility of education (**p.484-485**)

The estimates of the intergenerational correlation of education and earned incomes mentioned on p.484-485 mainly comes from Jantti-Jenkins 2013 (in particular 13, 16, 18), who themselves used the results from Bjorklund-Jantti 2009 and Bjorklund et al 2009. On the Swedish evolution, see Jantti-Jenkins 2013, figure 18. On the French evolution, see Lefranc 2011, figure 5. What is important in these estimates is to measure the average income over the lifetime (or at least on long period), otherwise we find much smaller correlations. For instance, Blanden-Gregg-Machin 2005 Table 2 gives figures ranging from 0.15 to 0.3. The same problem occurs with the estimates of Becker-Tomes. On the opposite, Lefranc 2011 finds IGE reaching up to 0.5-0.6 for France as soon as a real average income over the lifetime is computed. Also see the criticism by Bowles-Gintis 2002 of the estimates of Becker-Tomes.

Note on the inequality of access to the US universities (**p.485**)

The figures mentioned in the book on p.485 come from Duncane-Murnane 2011 (in particular figure 6.3 on the probability of access according to income). Also see the figures from Edsall 2012.

Note on the average income of the parents of Harvard students (**p.485**)

The data available on this question are rather limited. Generally, the most accessible data regard the incomes of the parents asking for grants or financial assistance (thus the bottom of the income distribution). We also have for every student, data on the parents' occupation, or on the income of the parents by income groups. See for instance Hoxby 2009 and Hoxby-Avery 2012. The difficulty is that we only have distributions by income groups and no exact amounts. We can then complete this data with fiscal data such as the one in Piketty-Saez 2003 on the evolution of average incomes in the different groups, in particular in the top of the distribution. We can also directly use fiscal data, either through the tax rebates thanks to the tuition fees, or by matching between the identification code for the parents and the children, which is in theory possible with comprehensive fiscal data, such as the ones recently used by researchers (for instance Chetty et al 2013). Combining these different approaches, we can estimate that the average income of the parents of the students in the universities of the Ivy League (like Harvard) is currently around $ 450,000, or approximately the average income of the wealthiest 2% American households.

Note on the average income of the parents of Sciences Po students (**p.779-780**)

This estimate is based on data sorted by income group of the parents published by Sciences Po following the implementation of a new system for the tuition fees. All the details on the calculations, as well as the detailed data by group, are available in this excel file. An important difficulty is the treatment of the data of the students coming from outside the EU, who all belong to the higher group, whatever their parents' incomes (their incomes are thus unknown, that's why I withdrew them from the main estimate). If these students are kept in the higher group, then we get an average income of 120,000/130,000 euros instead of 95,000 euros. I would like to thank Philippe Martin (Science Po) for the help he gave me to access these data.

Appendix to Chapter 14. Rethinking the Progressive Income Tax

Figure 14.1. Top income tax rates, 1900-2013 (**p.499**)
Figure 14.2. Top inheritance tax rates, 1900-2013 (**p.503**)
Figure S14.1. Top tax rate: "unearned income" vs "earned income"
Table S14.1. Top marginal income tax rate in rich countries, 1900-2013
Table S14.2. Top inheritance tax rate in rich countries, 1900-2013
(series used for figures 14.1-14.2 et S14.1)

The series used to construct figures 14.1-14.2, replicated in the book on p.499-503, as well as the supplementary figure S14.1, are available in tables S14.1-S14.2. All the details about the sources I used and the calculations I made are available in the corresponding excel file. As I explained in the book on p.500, the whole set of surcharges applying to the highest incomes (in the French case, the CSG has been excluded) has been included to compute the top income tax rate, except the surcharges for taxpayers who are single or married without kids three years after the wedding (taking into account these surcharges, the top income tax rate would have briefly reached 90% in France in 1925 and in 1940-1944; see Les hauts revenus en France..., 2001, figure 4.1 p.326; see the series here). For the United States and the United Kingdom, we separately give the rates for the “earned incomes” (labor income) and the “unearned income” (capital income), which, in these two countries, was taxed at a very high rate. To compute the top inheritance tax rate, only bequests to spouse and children were taken into account. In France as well as in Germany, other bequests have always been taxed a lot more. All the details are available in this excel file.

Note on the optimal tax rate for large on very high incomes (p.508-512)

I summarize on p.508-512, the main empirical results of the article "Optimal Taxation of Top Labor Incomes: A Tale of Three Elasticities", *American Economic Journal: Economic Policy*, 2013 (written with E. Saez and S. Stantcheva). I invite the interested readers to read the complete version available here for a comprehensive analysis of the results. In particular table 5 (p.52), explaining how it is possible to calculate the optimal tax rate on the highest incomes using the following formula:

$$\tau_L = (1 + tae_2 + ae_3)/(1 + ae)$$

where $e = e_1 + e_2 + e_3$ is the total elasticity of income
e_1 is the standard elasticity of labor
e_2 is the elasticity linked to income transfers between tax base (t is the tax rate of the least-taxed base) coming from the executives (or other economic agents)
e_3 is the elasticity linked to the effect of the negotiations for higher incomes from the executives (or other economic agents); this elasticity stems from the fact that the income is not always equal to the marginal product (for instance because the latter is not always easily observable)
a is the Pareto coefficient characterizing the top of the distribution

What needs to be emphasized is the following point. Even if only a limited part of the elasticity e is due to negotiation effects e_3, then it can significantly modify the usual results. For instance, for realistic parameters, the conclusion could be that the optimal marginal tax rate applying to the highest incomes is 82% instead of 57% (see table 5, p.52).

Note on the wealth of the American and French representatives (**p.514**)

The figures mentioned in the book on p.514 about the wealth of the members of the US House of Representatives come from data gathered by "Center for Reponsible Politics". This site gives the comprehensive list of the US Representatives, with lower and upper bound for their wealth. According to the US law, in order to declare their wealth, the Representatives must indicate to which bracket they belong to, and it is thus not possible to know the exact wealth level. In concrete terms, the brackets used are yet rather narrow, so that it has limited impact on the calculations of the average wealth. The figures mentioned, related to the members of the French governments, come from data published in 2013 on the website of the Prime Minister (here). Until recently, the French representatives do not publicly declare their wealth (the law changed in 2013, but the data is not available yet). As a consequence, it is not possible at this stage to make comparisons with the US Representatives.

Appendix to Chapter 15. A Global Tax on Capital

Note on the simulation of a European wealth tax (**p.515-530**)

I describe in the book on p.51-530 (in particular p.515-530) the revenues that can bring a European wealth tax, with different scales. The detailed results are presented in supplementary table S15.1, also available in the corresponding excel file.

Table S15.1. Basic simulation for a European wealth tax

Everyone interested can modify the parameters (in particular the tax rates and the tax brackets) in the excel file and immediately obtain the tax revenues and the corresponding number of taxpayers.

I present two variants in tables S15.1a and S15.1b. In variant a, the marginal rate of taxation is 0% for the wealth below 1 million euros, 1% between 1 million and 5 million euros, 2% above. The revenues would then be 1.8% of the European GDP (274 billion euros if the tax applies to the entire European Union).

In variant b, I add some marginal tax rates on the bottom brackets (0.1% for the wealth below 200.000 euros, 0.5% between 200.000 and 1 million euros) and the top brackets (5% between 20 and 100 millions euros, and 10% above). The revenues would then be 3.9% of the European GDP (582 billion euros if the tax applies to the entire European Union). What I have in mind is that a wealth tax such as variant b would aim at replacing all the other existing wealth taxes and in particular property taxes.

Everybody can modify the parameters as he wishes. All the details and the assumptions linked to these simulations are explained in the excel file. I would like to stress that this is just a basic simulator relying on simplifying assumptions, in particular on the shape of the wealth distributions. We can assume that the orders of magnitude are reliable in first approximation, but they should be refined. I want also to point out that the revenues only correspond to a tax that applies to everyone and its entire wealth without any special tax-exemption, and in particular without exemption for the wealth coming from businesses (this generally ends up withdrawing main part of the wealth from the tax-base at the top of the distribution, so that the wealth tax does no longer mean anything). If these scales would apply to

narrower definitions for the taxable wealth, then the revenues would be much more lower (refer to the discussion on the French wealth tax, ISF). Moreover, I did not try to take into account the wealth held by European citizens in tax havens. If some progress were made in this direction, then the revenues would be higher. Besides, the assumptions made on the shape of the Pareto coefficients at the top of the distribution also end up underestimating the potential revenues (this point is more precisely explained in this excel file).

Note on the formulas for the optimal capital taxation (**p.524-525**)

I mention in the book on p.525-525 (also on p.532) the formulas for the optimal capital taxation. The simplest case is the optimal inheritance taxation. Under some rather general hypothesis, it can indeed be demonstrated that the optimal inheritance tax rate (from the point of view of those who did not receive any bequest, that is around half of the population) is given by the following formula:

$$\tau_K = 1 - bG/y_L R$$

in which G is the cumulated growth rate over a generation, R is the cumulated return to capital over a generation, b measures the relative position of the zero-bequest-receivers individuals in the distribution of the bequest to the next generation (more precisely, the ratio between the average bequest that those who did not get any bequest plan to pass on their descendants, and the average bequest left by the whole population), and y_L measures the relative position of the zero-bequest-receivers individuals in the distribution of labor income (with a similar definition).[37]

This formula is analyzed in details in the article "A Theory of Optimal Inheritance Taxation", *Econometrica*, 2013 (written with E. Saez). In particular, in the extreme case $b/y_L=1$ (that is, those who did not receive anything hope to give as much as the average bequest of the population, or at least, as much as the part of the population having the same labor income), the formula simplifies to:

$$\tau_K = 1 - G/R$$

[37] This simplified formula $\tau_K = 1 - bG/y_L R$ corresponds to the case in which the long-term elasticity of capital supply toward the tax rate is equal to $e_K=0$ (savings and the ratio capital/income do not depend on the tax policy). In the general case, in which the elasticity can take any value, the formula can be simply written $\tau_K = (1 - bG/y_L R)/(1+e_K)$. If the elasticity tends to infinity (as the standard models with finite horizon implicitly assume), then the optimal capital tax rate is naturally equal to zero.

In other terms, in this case, the optimal inheritance taxation simply aims at compensating the effects due to the fact that the rate of return to capital is higher than the growth rate. If the growth were infinitely lower than the returns to capital then the interest of the ones who did not get anything would be to tax bequests at a rate close to 100% (even if they hope to leave as much as the others). The intuition behind is explained in the article mentioned above, p.7. Also see figures 1-2 of this article, which show some simulations for the optimal rates in France and in the United States, given the observed distributions of the bequests passed on to descendants and the received bequests.

Generally, the formulas for the complete system of optimal capital taxation are more complex than the above formula, because the optimal tax system includes not only a progressive inheritance tax, but also an annual tax on capital incomes and capital stock, depending in particular on the volatility of the returns to capital in the future and on the elasticity of the latter. For a detailed analysis of the different effects, see "A Theory of Optimal Capital Taxation", NBER 2012 (written with E. Saez, longer version of the previous article).

Note on the debate on the capital tax (**p.526**)

I write in the book on p.526 that the debates on the capital tax were often characterized by extreme positions in the history: either it was massively rejected, or considered as the magic tax that will replace everything, and in particular replace the income tax (at least a substantial part), which is not really realistic. For instance, see the famous proposal made My Maurice Allais in his book of 1977 - *L'impôt sur le capital et la réforme monétaire* (The capital income and the monetary reform) and also Allais 1982. We also find in numerous countries and periods this kind of debates about the “unique capital tax”, coming from a wide and diverse range of political and intellectual thoughts.[38]

Note on the data from ISF (**p.528**)

I mention on p.528 the limitations and the irregularity of the French tax administration data during the years 2000-2010 regarding the ISF (for example, compared to

[38] For instance, check the campaign for the « TNA » (Tax on net assets) in France currently conducted by the «Comité Bastille» (and also A. Teissier du Cros, *La France, le bébé et l'eau du bain*, L'Harmattan 2009. Check as well, the proposal for the "fair share taxes" developed in the United States by the Fair Share Taxes network. In the US case, the aim is not to replace the income tax but to complete it. This proposal is in the same view.

inheritance data published by the same administration between 1900 and 1960). Zucman 2008 attempts to gather and to coherently treat the statistics available on the ISF. The main conclusion is that the evolution of the wealth that is reported during the 1990s-2000s is consistent the general evolution of wealth in France. However, this does not imply that the ISF is properly operating (given the numerous exemption and strategies to avoid it, the wealth that is declared is much lower than the actual wealth).[39] This merely suggests that the situation has not deteriorated over time. It should be noted that the number of households affected by the ISF was 600,000 in 2011 (or around 2-3% of the adult population, including spouses), and should fall down to 250,000-300,000 in 2012-2013, given that the threshold for the taxable wealth rose from 800,000 euros to 1.3 million euros (after the deduction of all the tax-exempt assets, and after the deduction of the 30 % allowance for the main residence).

Note on the capital tax in Sweden and in Spain (**p.533**)

I give on p.533, some information on the evolution of the capital tax rate in several European countries (in particular, Sweden and Spain). On the evaluation of tax rates and tax bases for the capital tax in Sweden from the creation until its abolition in 2007, refer to S. Hotchguertel, H. Ohlsson, "Who is at the top? Wealth mobility over the life cycle", Uppsala Universitet WP, 2012, Tables B1-B2 and voir. G.D. Ritez, M. Henrekson, D. Waldenstrom, "The Swedish Inheritance and Gift Taxation, 1885-2004", Uppsala Universitet WP, 2012 and also H. Ohlsson, "The Legacy of the Swedish Gift and Inheritance Tax, 1884-2004", *European Review of Economic History*, 2011. On the evolution of the wealth tax in Spain, refer to Alvaredo-Saez 2009. On Switzerland, refer to this article. On the debates about the creation of a wealth tax in the United Kingdom, refer to Glennerster 2011.

Note on the education spending in Egypte (**p.538**)

I write on p.538 that the budget for education in Egypt is smaller than 5 billion dollars, This simply comes from the fact that the GDP of Egypt is around 250 billion dollars (current exchange rate, that is 500 billion dollars at purchasing power parity), and that the spending on education account for about 1-2% of GDP (World Bank estimates, with some variations depending on the year).

[39] On this specific point, check the simulations on the website révolution fiscale to know what would be the revenue of a wealth tax applied on the entire wealth (without exemption).

Note on the migration rates in Europe and in the United States (**p.538-539**)

For an analysis on recent migration statistics, read for instance the op-ed of Mouhoub 2012. The article Ashenfelter 2012 (which shows that the purchasing power can vary from 1 to 10 for being a waiter at McDonald's in different countries) gives a pretty good summary of the magnitude of the migratory pressure. For a recent analysis on the debates about migration policies in Europe, refer for instance to *Europe's Immigration Challenge: Reconciling Work, Welfare and Mobility* (By Elena Jurado & Grete Brochmann, Policy Network 2013)

Appendix to Chapter 16. The Question of the Public Debt

Note on the balance sheet of the central banks (**p.550-553**)

I describe on p.550-553 the evolution of the balance sheet of the central banks since 2007-2008, and I place it into an historical perspective. To consult the detailed series on the structure of these balance sheets, see *Capital is Back...*, 2013 (in particular Table UK.7, Table FR.7, etc.).

Note on the concept of the golden rule (**p.563**)

I present on p.563 the concept of the "modified golden rule" $r = \theta + \gamma g$ of capital accumulation (see the discussion about this formula on chapter 10). In particular, I write on p.564 that the equality r = g corresponds to an upper bound of the desirable accumulation. The main intuition is the following. Above the level given by the golden rule (with a return lower than the growth) the capital share would be smaller than the savings rate in the long run: this would be an absurd situation for everybody, since maintaining the capital at this level would require to allocate every year more resources to investment than what the capital yields. This type of "dynamic inefficiency" can arise if the individuals save without taking into account the returns, for instance for their old age, and if their life expectancy is long enough. In this case, the efficient policy consists in reducing the capital stock, for example issuing public debt (in large quantity if needed) and replacing de facto the funded pension scheme by a "pay as you go" pension scheme. Yet, this interesting theoretical possibility has never been put into practice: in the societies we know, the average return to capital is always higher than the growth rate. These questions of "dynamic efficiency", and the links with the question of optimal capital taxation are more precisely explained in "A Theory of Capital Taxation", 2012, p.83-104. Also refer to the well-known article Phelps 1961, as well as Allais 1982.[40]

[40] It is interesting to note that Allais (who wrote at the end of "The Glorious Thirty") had in mind extremely high growth rate, like g=4%, such that the rate of return on capital around r= 5-6% seemed close to the « capitalist optimum r=g ». It is not same with a smaller growth rate. Furthermore, he considered that the capital/income is not very elastic towards modification on taxes (because accumulate wealth for very different purposes), and that a wealth tax would have little effect on the wealth accumulation (and moreover would enhance the incentives to produce, contrary to the income tax). See further discussions in chapter 15.

Note on the primary and secondary public deficit (**p.567**)

I write in the book on p.567 that the interest payments on the government debt often account for much a bigger part than the primary deficit, which has been small or zero in most of the rich countries over the last decades. The following table, taken from *Capital is Back...*, 2013, give a complete decomposition of the structure of the public deficit and of public savings in rich countries over the period 1970-2010. All the details are available in the corresponding excel file.

Table S16.1. Government savings vs deficits in rich countries, 1970-2010

Note on the golden rule-related questions applying to climate change (**p.567-569**)

I summarize on p.567-569 the controversy opposing Stern 2006 to Nordhaus 2007 about the discount rate to be applied to the future damages due to climate change. (I also mentioned the papers of Guesnerie 2004, 2010 and Sterner 2008).

More precisely, Stern like Nordhaus, both use the modified golden rule, that is the formula $r = \theta + \gamma g$. They both assume that θ=0,1% and g=1,3%. But they make totally different hypothesis on the concavity parameter γ. Stern assumes γ=1, hence r=1,4%. Nordhaus assume γ=3, hence r=4%. For a given evaluation of the future damages due to climate change, it dramatically modifies the conclusions for the present. For further precisions, refer to these lecture notes.

Note on the informational and constructivist approach to political institutions **(p.569)**

See this survey.

Note on the “compendium of pensions” published in 1789 -1790 (**p.570**)

The “Nominative state of pensions on the Royal Treasury” ("Etat nominatif des pensions sur le Trésor royal"), or often called more simply “compendium of pensions” (“Grand livre des pensions”), is a list by names and amounts, of which the Assemblée Nationale ordered the release in 1789, and which was eventually printed on April 21, 1790. It is replicated in the *Archives parlementaires* published by Mavidal-Laurent, 1st series tomes 13 (p.301-772), 14 (p.1-812) et 15 (p.1-244). It contains more than 23,000 names with their respective pensions.

Note on the accounts of the Lonmin, Inc. (**p.570)**

As I write in the book on p.570, it is not easy in practice to analyze how the value added can be divided between wages and profits in a private company, because the accounts published by those companies often group wages and intermediate consumption in the same line. For example, the profit account of 2011 published on the website www.lonmin.com, gives us the following information. In 2011, Lonmin, which employs around 25,000 workers, mainly miners and engineers on the site of the giant mine Marikana, had total sales around 1.9 billion dollars (I have rounded figures to simplify). We know the total costs are 1.5 billion dollars, to be deducted from the sales of 1.9, hence the profits of 400 million dollars. But how the total costs can be split between wages and intermediate consumption is not mentioned anywhere. Using the available data on the wages structure and the social contribution rates available on the website. We can estimate that the wage bill of the 25,000 workers is around 500 millions dollars (including social contributions) and thus the intermediate consumption accounts for about one billion dollars. In this case, we have for 2011 around 900 billion dollars of value added approximately dividing in 500 million for the wage bill and 400 million for gross profits (around 300 million for the net profits, after deducting capital depreciation). In other words, the share of the gross profits in the gross value added is 44% (and 37% for the net profits).[41] The accounts published by Lonmin also a show that the capital was worth 3 billion dollars, thus a profit rate of 10%. But it is only approximate, because the exact data necessary to make these calculations are not publicly accessible. It is a common problem of the accounts of the companies in the world, which prevents from calculating the capital-labor split. Besides, it would be desirable to publish the detailed breakdown of the wage bill by wage brackets, detailed breakdown of purchases by expenditures categories (in order to spot the potential abuses, for instance like accommodating expenditures – luxurious hostels, business class air tickets, etc. – aiming at completing the revenue of executive, which apparently is quite widespread), and so forth.

[41] 400/900 = 44%, et 300/800 = 37%.

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